SAMPLE LESSON: MATHEMATICS

Class: Form 4

Title of Module: Plane Geometry	**Title of Chapter:** Transformation of plane shapes

Title of Lesson: Rotation

Duration of Lesson: 90mins

Module: Plane Geometry; **Topic:** Transformation of plane shapes

Lesson 4: Rotation

Objectives:

In these lessons, you will learn

- What a rotation is;
- How to draw the rotated image of an object given the center, the angle and the direction of rotation;
- How to find the angle of rotation given the object, its image and the center of rotation;
- How to rotate points and shapes on the coordinate plane about the origin.

Rationale:

Rotations as well as other transformations are everywhere around us. Rotation is used to create designs by architects as they take models and turn to form patterns, we see them even in the tiling of floors and other surfaces in general. We also see situations of rotation in things such as merry-go-round in playing areas for children, fans, wheels of vehicles and bicycles, kitchen blenders, steering wheel of a car etc. the earth experiences one complete rotation on its axis every 24 hours.

Didactic material you need for this lesson:

Grid, Square paper or graph paper
Pencil, ruler, set square, protractor

Prerequisite knowledge: You can do the following:

- Identify different plane figures;

- Plot points and read out coordinates of a point;

- List properties of congruent plane figures;

- List properties of similar figures;

- Establish that two plane figures are similar;

- Measure distances and angles or calculate using Pythagoras theorem.

References:

1. GEOMETRY, Eugene D. Nicholas, Mervine L. Edwards, E Henry Garland, Sylvia, A Hoffman, Albert Mamary, William F Palmer (1991), Holt, Rinehart and Winston, Inc.
2. https://www.mathwarehouse.com/transformations/rotations-in-math.php
3. https://www.mathsisfun.com/geometry/rotation.html
4. https://www.onlinemathlearning.com/rotation-transformation.html
5. https://study.com/academy/lesson/rotations-in-math-definition-lesson-quiz.html
6. https://www.bbc.co.uk/bitesize/guides/z9jpv9q/revision/4

Verification of pre-requisite knowledge

1. Give the name of each of the figures below:

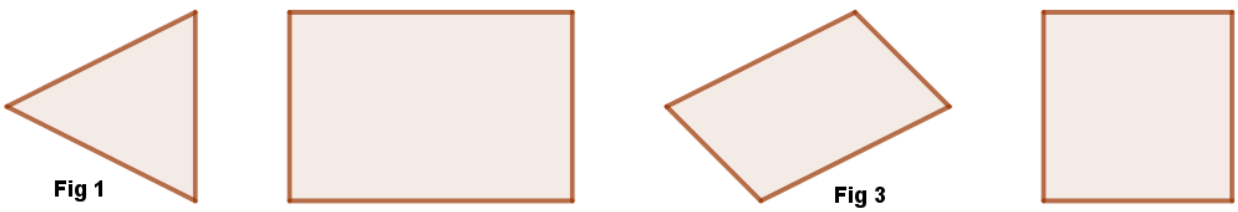
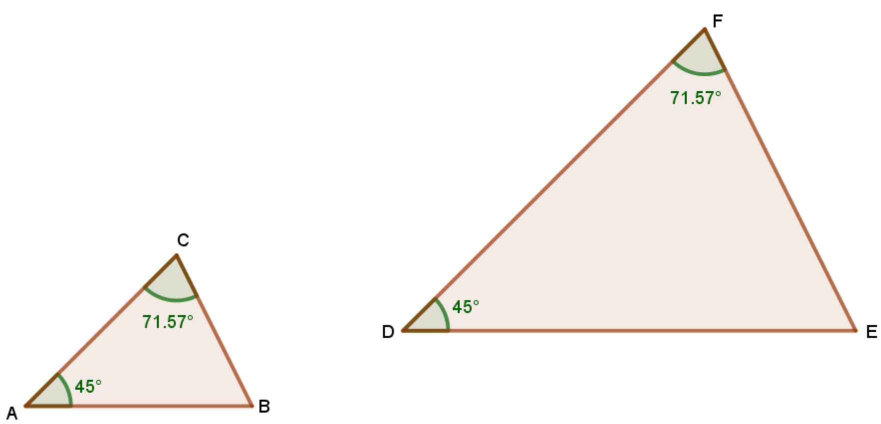


Fig 1 Fig 2 Fig 3 Fig 4

2. Given the points A(1, 3); B(8, 4); C(0, 3) and D(4, 6), use any method of your choice to find the distances AB; BC and AD.
3. Are the two triangles below congruent or similar? Justify.



Introduction

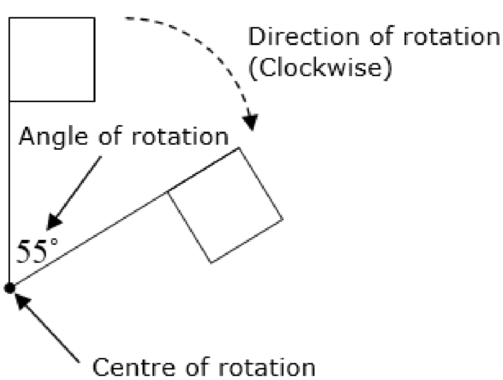
A **rotation** is a transformation that turns a figure/object about a fixed point through a given angle in a defined direction. The **Direction** of **Rotation** can be **clockwise** or **anticlockwise**. The fixed point in which the rotation takes place is called the **center of rotation.**

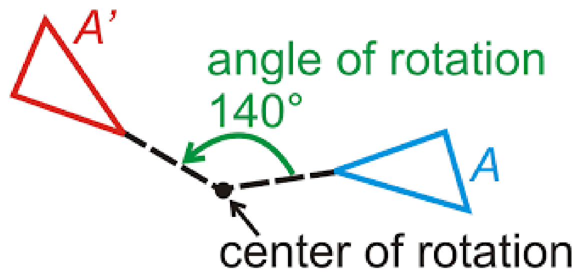
An object and its **rotation** are the same shape and size, but the figures may be turned in different directions. The distance from the centre to any point on the shape stays the same. The amount of rotation made is called the **angle of rotation**.

For any rotation you need:

- An object (point, line, plane figure)
- A fixed point (called the centre of rotation)
- Angle of rotation
- direction (either clockwise or counterclockwise)

Example:

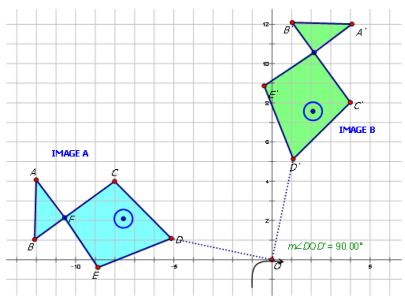


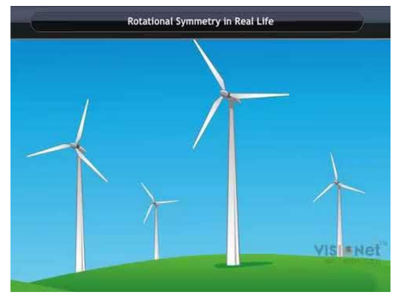


Notation:

The Mathematical notation for a rotation is usually written as **R(Centre, Rotation)**, where the centre is the point of rotation and the rotation is the amount of turn given in degrees.

Images of some Rotation



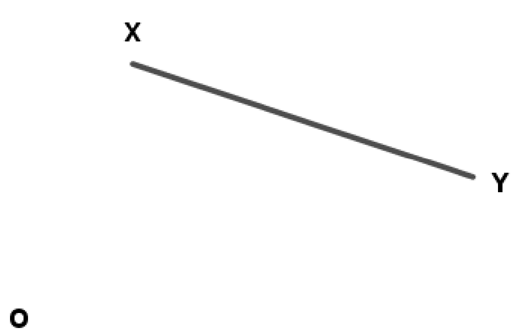



Drawing the Rotated Image

Given an object, the center of rotation and the angle of rotation we can determine the rotated image of an object.

Example 1:

Determine the image of the straight line segment XY under an anticlockwise rotation of 90° about O.

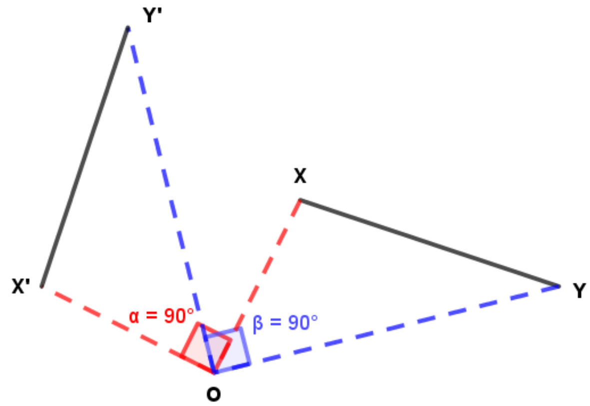


Solution:

Step 1: Join point *X* to *O to form line OX.*

Step 2: Using a protractor, draw a line 90˚ anticlockwise from the line *OX.* Mark on the line the point *X'* such that the line of *OX* = *OX'*

Step 3: Repeat steps 1 and 2 for point *Y.* Join the points *X'* and *Y'* to form the line *X'Y'.*

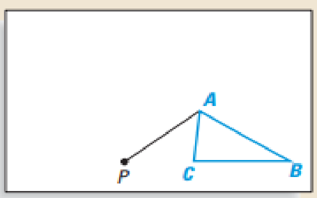


Construction Image of a figure by rotation

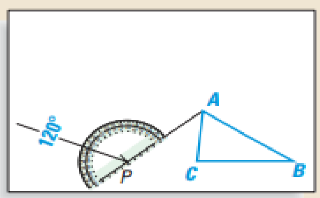
Construction

Rotating a Figure

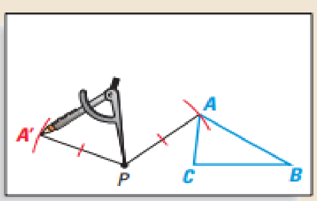
Use the following steps to draw the image of △*ABC* after a 120° counterclockwise rotation about point *P*.



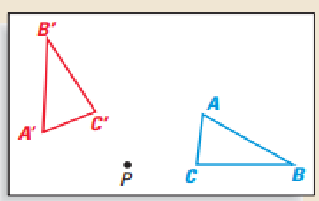
1. Draw a segment connecting vertex *A* and the center of rotation point *P*.



2. Use a protractor to measure a 120° angle counterclockwise and draw a ray.



3. Place the point of the compass at *P* and draw an arc from *A* to locate *A'*.



4. Repeat **Steps 1–3** for each vertex. Connect the vertices to form the image.

The Angle Of Rotation

Given an object, its image and the center of rotation, we can find the angle of rotation using the following steps.

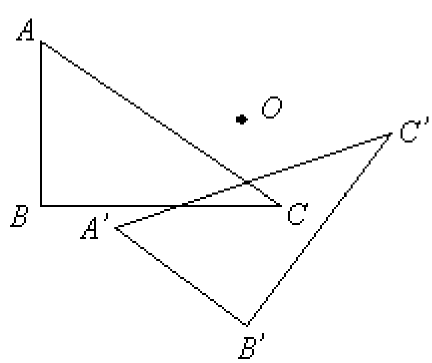
Step 1 : Choose any point in the given figure and join the chosen point to the center of rotation.

Step 2 : Find the image of the chosen point and join it to the center of rotation.

Step 3: Measure the angle between the two lines. The sign of the angle depends on the direction of rotation. Anti-clockwise rotation is positive and clockwise rotation is negative.

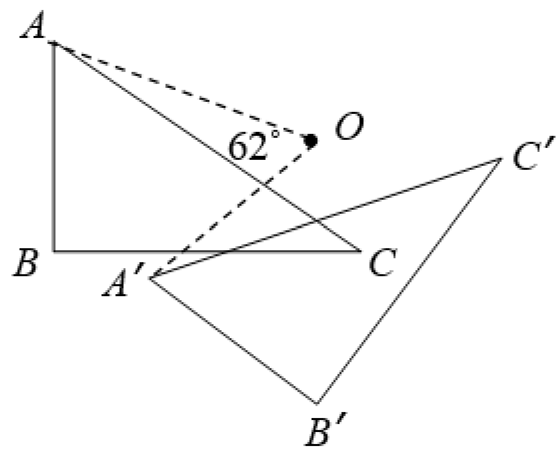
Example

Figure *A'B'C'* is the image of figure *ABC. O* is the center of rotation. Find the angle of rotation.



Solution :

Step 1: Consider point A and its image A’
Step 2: Join A to O
Step 3: Also join A' to O.
Step 3: Measure the angle AOA'



The angle of rotation is 62˚ anticlockwise or +62˚

Rotation of Points and shapes on the coordinate axis.

You are now going to see how points and shapes are rotated on the coordinate plane. It will be helpful to note the patterns of the coordinates when the points are rotated about the origin at different angles.

Rotation of a polygon about a given point through a given angle

You will need a straightedge, a protractor, and a compass.
You are given:

- A center of rotation.
- A figure to rotate.
- An angle of rotation.

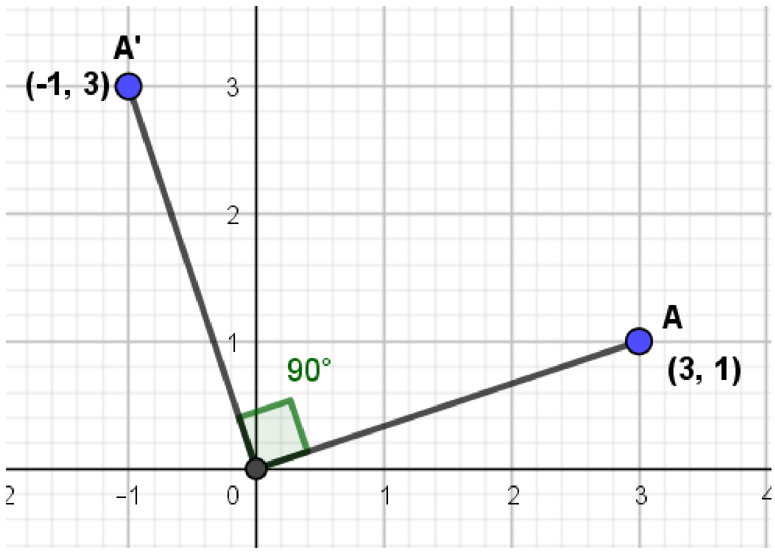
Steps to follow in order to rotate a figure around a fixed point using a compass and protractor

1. Draw a ray from the center of rotation to the point you wish to rotate.
2. Draw an angle with the center of rotation as the vertex.
3. Use a compass to draw a circle (arc) with the center at the center of rotation and a radius from the center of rotation to the point you are rotating.
4. Now rotate all the other points and connect the dots.

Specific Rotations

I- Rotation by 90° about the origin: $R_{(origin,\ 90°)}$

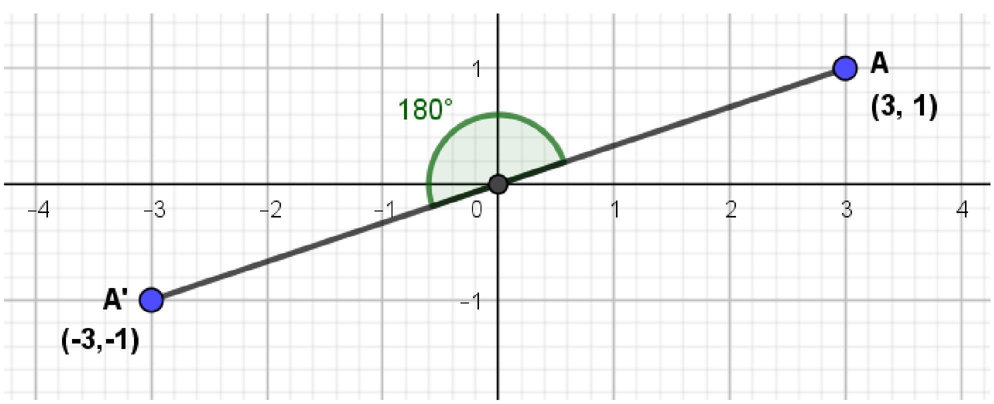
A rotation by 90° about the origin anti-clockwise can be seen in the picture below in which A is rotated to its image A'. The general rule for a rotation by 90° about the origin is (x, y) ---→(-y, x).



In the above image, the point (3, 1) has been rotated by 90° about the origin anti-clockwise, to the point (-1, 3).

II- Rotation by 180° about the origin: $R_{(origin,\ 180°)}$

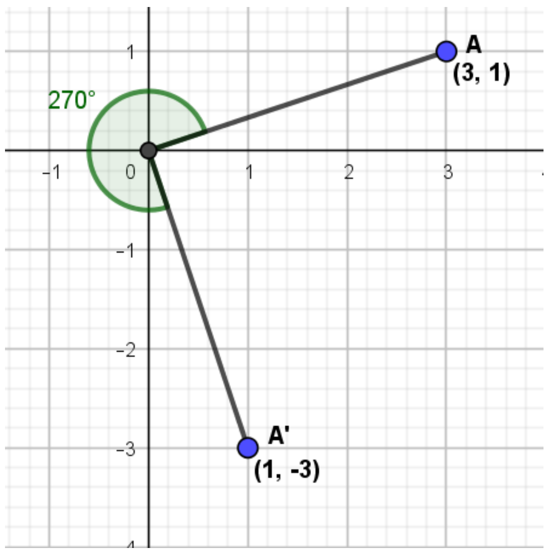
A rotation by 180° about the origin anti-clockwise can be seen in the picture below in which A is rotated to its image A'. The general rule for a rotation by 180° about the origin is (x, y) ---→(-x, -y).



From the above image the point A(3, 1) has been rotated by 180° about the origin to the image point A’(-3, -1). For 180 degrees, the rule is (x, y) --------> $(-x, -y)$

III- Rotation by 270° about the origin: $R_{(origin,\ 270°)}$

A rotation by 270° about the origin anti-clockwise can be seen in the picture below in which A is rotated to its image A'. The general rule for a rotation by 270° about the origin is (A,B) ---→(B, -A).



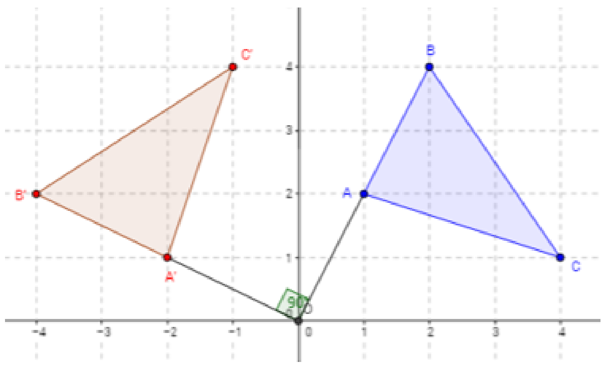
The point A (3, 1) has been rotated by 270° about the origin to the image point A’ (1, -3).

For 270 degrees, the rule is:

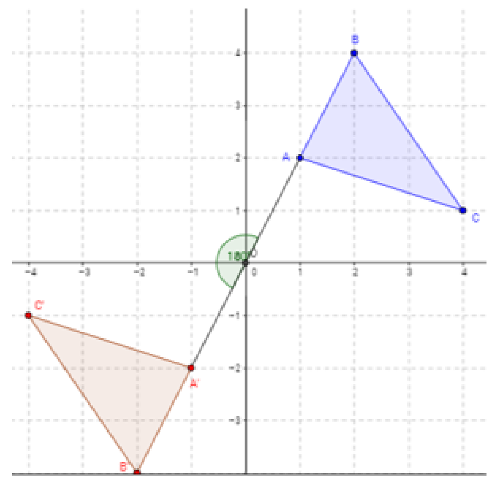
(x, y) --------> $(y, -x)$

Summary images of rotation by common angles about the origin, anti-clockwise.

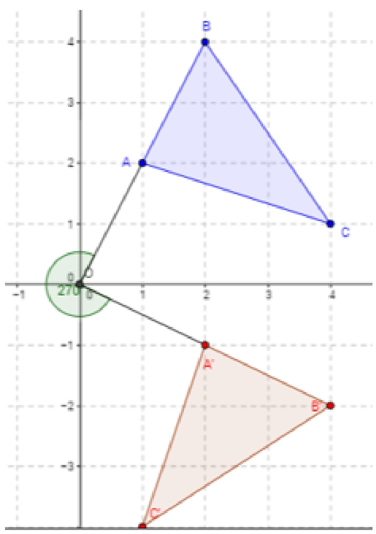
Rotation about the Origin



Rotation of 90 degrees
R(x, y) = (–y, x)

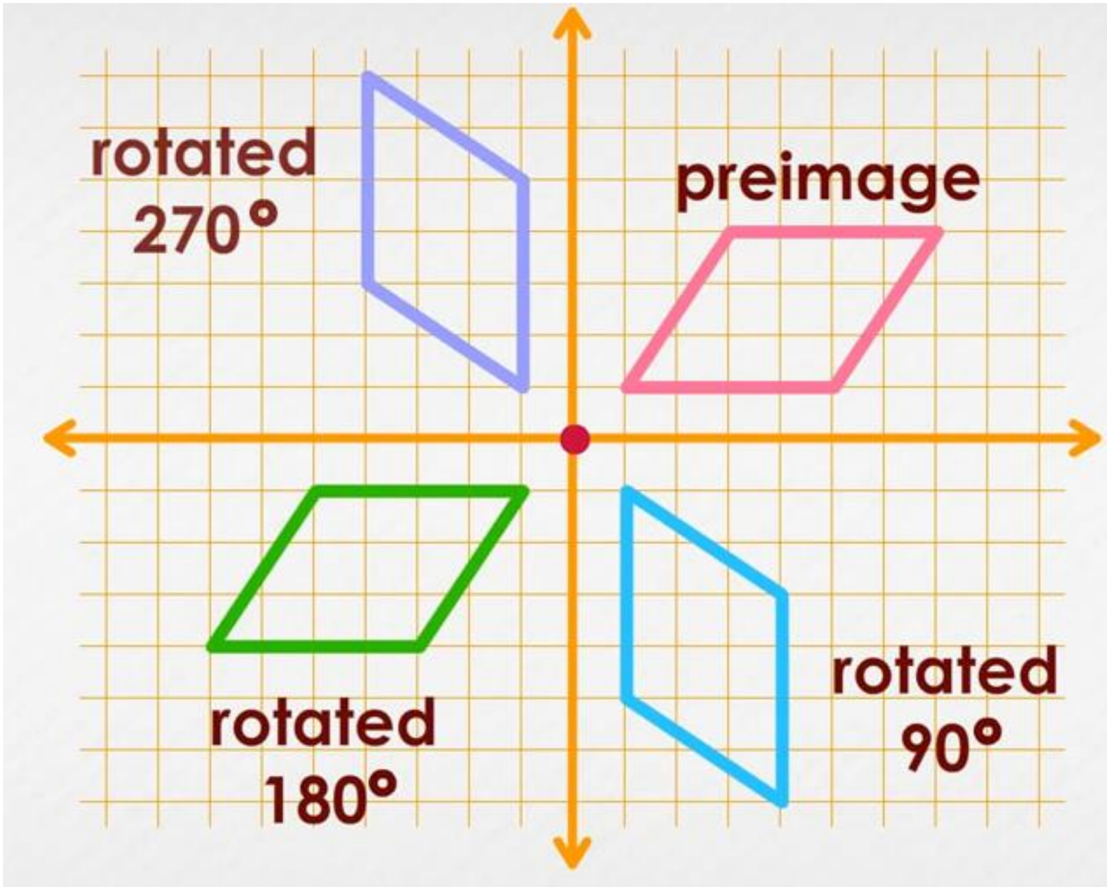


Rotation of 180 degrees
R(x, y) = (–x, -y)



Rotation of 270 degrees
Or Rotation of – 90 degrees
R(x, y) = (–y, -x)

Images of rotation by common angles, clockwise about the origin.



Point to Remember:

For any rotation, we need to specify the center, the angle and the direction of rotation.

Center of Rotation is the point around which you are performing the rotation. This point is fixed.

The angle of rotation is the amount of turn the object has to go through.

A clockwise rotation is a negative magnitude and a counterclockwise turn has a positive magnitude.

The **distance** from the centre of rotation to any point on the shape stays the same.

A rotation is an isometric transformation: the original figure and the image are congruent.

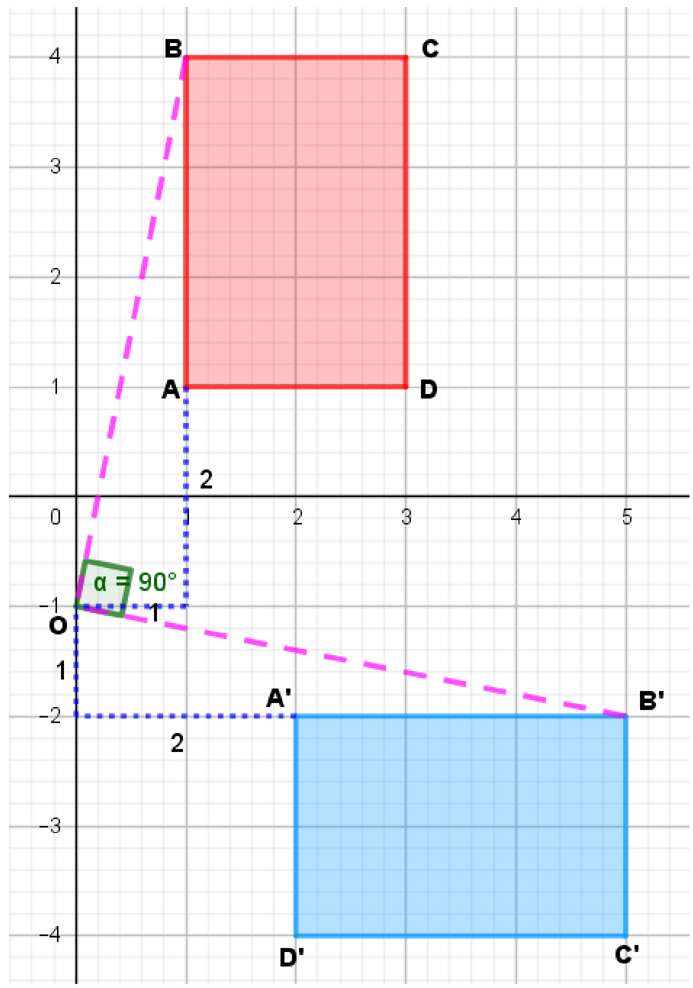
The figure will not change size or shape, but, will change direction.

At times, the centre of Rotation can be different from the origin.

Example:

Rotating rectangle ABCD with A(1, 1); B(1, 4); C(3, 4) and D(3, 1) through 90^{o} clockwise about the point (0, -1) as centre.

The line that connects the object point to the centre of rotation is perpendicular to the line that connects the corresponding image point to the centre of rotation. Each corner of the image rectangle A’B’C’D’ is the same distance from the centre of rotation as the original corner of the object rectangle ABCD.



Homework:

1. a) In the coordinate plane, sketch the triangle A(2, -2), B(4, 1) and C(5, -1), to form triangle ABC.

b) Rotate triangle ABC by 90^o counterclockwise about the origin. Name the figure A'B'C'.

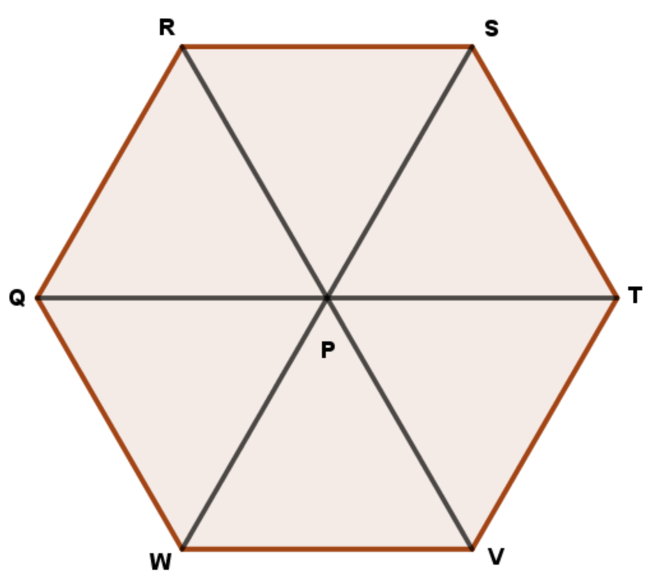
2. Consider the points A(1, 1); B(5, 2); C(4, 4,); A'(-1, 1); B'(-2, 5) and C'(-4, 4). A rotation of 90^o about the origin rotates ABC to A'B'C'.

a) Is the rotation clockwise of counterclockwise or clockwise?

b) Is AB = A'B'?

c) Is AA' = BB'?

3. The diagonals of the regular hexagon form six equilateral triangles. Use the diagram to complete the sentences that follow.



a) A clockwise rotation of 60^o about P rotates R onto _______

b) A counterclockwise rotation of 60^o about about ____maps R onto Q.

c) A clockwise rotation of 120^o about Q maps R onto ________

d) A counterclockwise rotation of 180 about P rotates V onto.