SAMPLE LESSON: MATHEMATICS

Class: Form 4

Title of Module: Plane Geometry	**Title of Chapter:** Transformation of plane shapes
Lesson: Simple Transformation(Dilation, Translation)	

Class: Form 5

Topic: Plane Geometry

Lesson: Transformation of plane figures

Objectives of Topic: At the end of the lesson, you should be able to:

- List different types of transformation

- Describe each type of transformation

- Find image of plane figures for any transformation, geometrically or by use of a matrix operator.

Prerequisite knowledge: You can do the following:

- Identify different plane figures;
- Plot points and read out coordinates of a point;
- List properties of congruent plane figures;
- List properties of similar figures;
- Establish that two plane figures are similar;
- Measure distances and angles or calculate using Pythagoras theorem.

If you cannot do any of the above, go back and revise these notions.

Motivation: Transformation is all around us and is experienced or used in many real life situations. The architects, use transformations to include decorative patterns and designs in their plans for a building. Some other real life examples of transformations are the movement of an aircraft as it moves across the sky, sewing with a sewing machine, throwing a shot-put are all examples of transformation.

Didactic Materials:

Pictures illustrating different patterns of transformation in the environment (2) graph papers or square papers.

References:

1. GEOMETRY, Eugene D. Nicholas, Mervine L. Edwards, E Henry Garland, Sylvia, A Hoffman, Albert Mamary, William F Palmer (1991), Holt, Rinehart and Winston, Inc.
2. *Mathematics 9*, M.J. Tipler, J Douglas (2004), Nelson Thornes Ltd
3. https://tutors.com/math-tutors/geometry-help/transformations-in-math-definition-examples;
4. https://study.com/academy/lesson/dilation-in-a-coordinate-plane.html

Introduction

Transformation involves moving an **object** from its original position to a new position. The object in the new position is called the **image**. Each point on the object is mapped to another point on the image.

In these lessons, we will study the following types of transformations in math:

Dilation which involves a resizing of the object. It could result in an increase in size (enlargement) or a decrease in size (reduction);

Translation which involves "sliding" the object from one position to another;

Reflection which involves "flipping" the object over a line called the line of reflection;

Rotation which involves "turning" the object about a point called the center of rotation;

Shear which involves fixing all points along one side of a preimage while all other points of the preimage move parallel to that side in proportion to the distance from the given side; "a skew.,"

Translation, reflection and rotations are called **isometric transformations** (**rigid transformations**) because the image is the same size and shape as the original object. The original object and the image are **congruent**.

Dilation is not an isometric transformation (**non-rigid transformation)** because the size of the image is not the same as the object. The original object and the image are **similar** i.e. they have the same shape but different size.

Shear is not an isometric transformation because the shape of the image is not the same as the shape of the object. The original object and the image have the same area but not the same shape.

Each of the different types of transformation will be treated as a lesson.

Lesson 1: Dilation (Enlargement, Reduction)

Objectives: At the end of this lesson you should be able to:

- Describe a dilation;
- Identify situations of dilation in real life;
- Find image of a point, a line a plane figure under translation;
- Perform or illustrate dilation on the coordinate system;
- Give the scale factor for a dilation;

Motivation:

Have you taken a picture on your phone and ordered a print of it? Is that print the same size as the picture on your phone, or is it larger or smaller?
Objects like pictures can be dilated. Objects in the coordinate plane can also be dilated as we will see in this lesson.

Didactic material (what you need for the lesson)

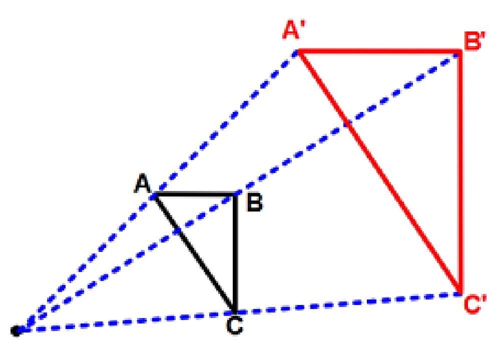
Pencil, pen, ruler, Square paper or graph paper

Introduction

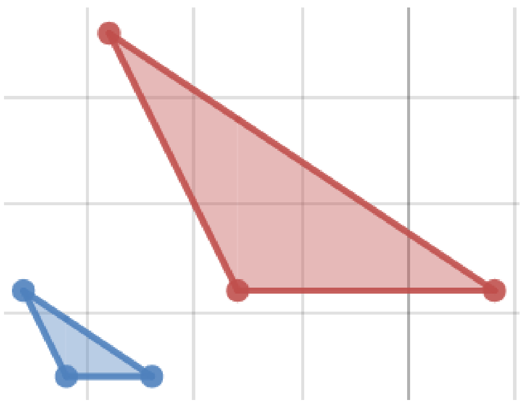
A dilation is a transformation that enlarges or reduces the size of the image but without changing the shape and orientation of the object.

This lesson illustrates a dilation in a coordinate plane, by demonstrating how to use the origin as the center of the dilation and the given scale factor to find the coordinates of the vertices of the image

Look at the figures below:



The two triangles ABC and A’B’C’ are similar. They have the same shape and the same orientation. They do not have the same size. Triangles ABC is enlarged to have triangle A’B’C’. If ABC is the object, then A’B’C’ is the image.
Triangle A’B’C’ is reduced to have triangle ABC. If A’B’C’ is the object, then ABC is the image.



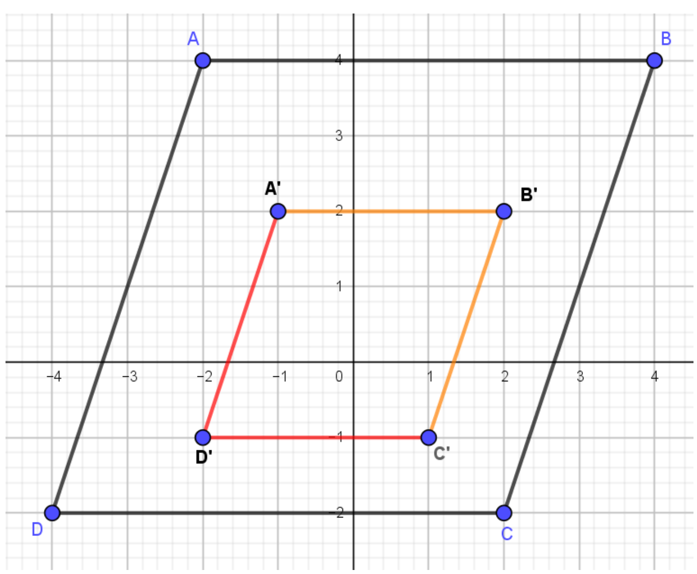
The two triangles have the same shape and orientation.

They do not have the same size. One is bigger than the other.

One is the image of the other by a dilation.

Activity 1

Consider the figure below and carry out the activities that follow



Instructions:

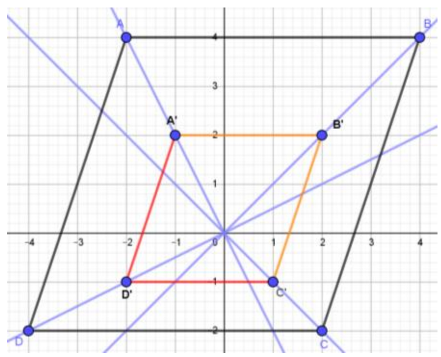
1. Write out the coordinates of the vertices of A, B, C and D
2. Write out the Coordinates of the points A', B', C', and D'
3. Divide the respective coordinates of the points A, B , C , and D by 2

4. Compare your result with the coordinates of the points A’, B’, C’, and D’ respectively
5. What can you say is the relationship between the coordinates of each corresponding points?
6. Take your graph paper on draw these plane figure.
7. Draw staight lines connecting each point and its image point. What is the coordinates of the point of intersection of these lines?
8. Using Pythagoras theorem to determine the distance of A from the point of intersection of these points. Also find the distance of A’ from this same point of intersection.
9. Repeat VIII for B and B’, C and C’, D and D’
10. Consider ABCD to be the object and A’B’C’D’ to be the image. What is the relationship between distance of object points and distance of image points?

Solutions to Activity:

1. A(-2, 4); B (4, 4); C (2, -2) and D(-4, -2).
2. A’(-1, 2); B’(2, 2); C’ (1, -1) and D’(-2, -1).
3. $\left(\frac{-2}{2}, \frac{4}{2}\right) = (-1, 2)$; $\left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$; $\left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$ and $\left(\frac{-4}{2}, \frac{-2}{2}\right) = (-2, -1)$.
4. The coordiates are those of A’, B’, C’ and D’ respectively.
5. The coordinates of each point of figure A’B’C”D’ is half the coordinates of the corresponding point of figure ABCD.
6. Diagram as above.

7.



The blue lines join corresponding points.
All these four lines intersect at the point with coordinate (0, 0)

8. $OA = 2\sqrt{5}$, $OA' = \sqrt{5}$
9. $OB = 4\sqrt{2}$, $OB' = 2\sqrt{2}$; $OC = 2\sqrt{2}$, $OC' = \sqrt{2}$; $OD = 2\sqrt{5}$, $OD' = \sqrt{5}$
10. Distance of each image point is half the distance of the corresponding object point.

Summary

Following the activity, the figure ABCD is the object and the figure A’B’C’D’ is its image by a Dilation. The shape of ABCD and that of A’B’C’D’ are the same, their orientation the same but the object is bigger than the image.

The size of the image is half the size of the object.

We say the **Scale Factor** for this dilation is $\frac{1}{2}$.

The **Scale Factor** determines how much larger or smaller the image will be as compared to the image.

A sclae factor **< 1** makes the **image smaller** while a scale factor **> 1** makes the **image bigger**.

To find the dilated image, we first must know the coordinates of our original image. Then we simply multiply the coordinate by the scale factor to find the dilated image.

Where the dilation object is drawn will not only depend on the scale factor, but also where the center of dilation is situated.

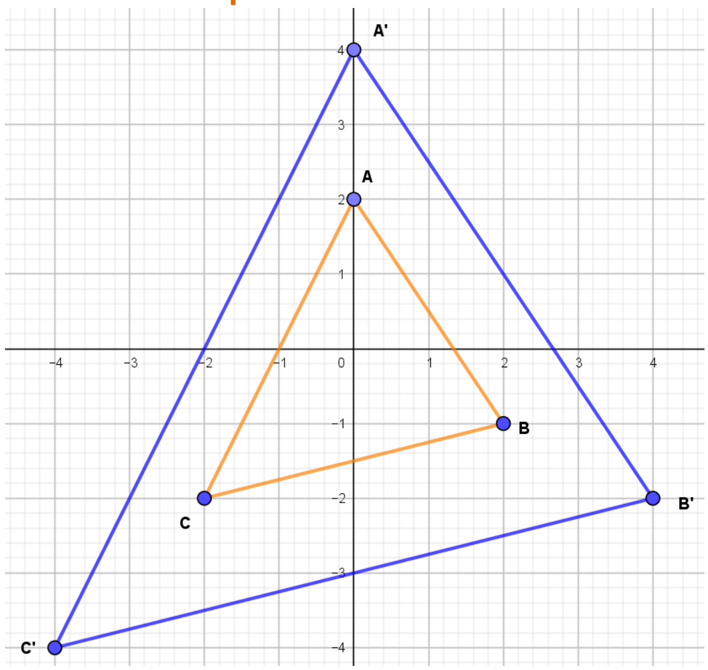
When corresponding points of the object and image are connected by straignt lines, they will meet at a point. In the above example, they all meet at the point with coordinate **(0, 0).** This point is call ed the **Centre of the Dilation.** This centre can be any other point on the coordinate plane.

From the activity, distance of A' from the centre is half the distance of A from the centre.

Similarly, $OB' = \frac{1}{2}OB; \quad OC' = \frac{1}{2}OC \quad and\ OD' = \frac{1}{2}OD$

The distance of the image point from the center of dilation is got by multiplying the distance of the object by the **Scale Factor**.

Example 1:

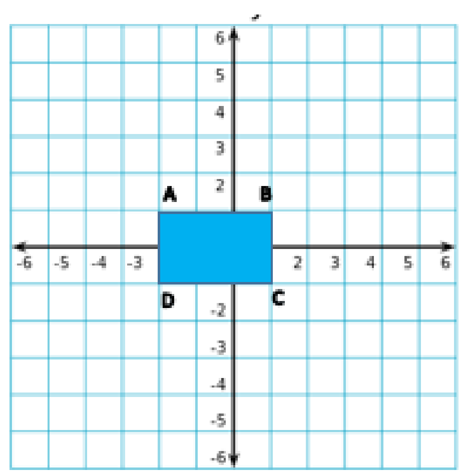


Tringle ABC is the object and triangle A'B'C' is the image. A(0, 2); B(2, -1); C(-2, -2)
and A'(0, 4); B'(4, -2) and C'(-4, -4)
The coordinate of each point is multiplied by 2 to have the coordinate of the corresponding image point.
Therefore, the **Scale Factor is 2**.
The centre of this dilation is the origin (0, 0).
The distance of the point A from the center is 2cm and the distance of its image A', from the centre is 2 × 2= 4cm.
This will be same for all the other points.

Example 2:

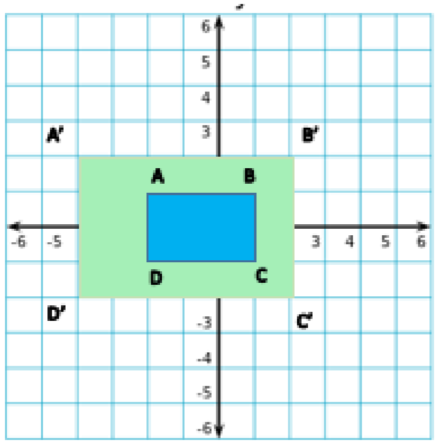
Given rectangle ABCD with coordinates A(-2, 1), B(1, 1), C(1, -1), D(-2, -1).
Dilate ABCD by a scale factor2 with the origin as center of dilation.

Plotting the different points we have the figure beside



Multiply each coordinate by 2 to have the corresponding image points; doing this will give: A'(-4, 2), B'(2, 2), C'(2, -2), D'(-4, -2)

Graph the image figure with the original image on the same graph to have:



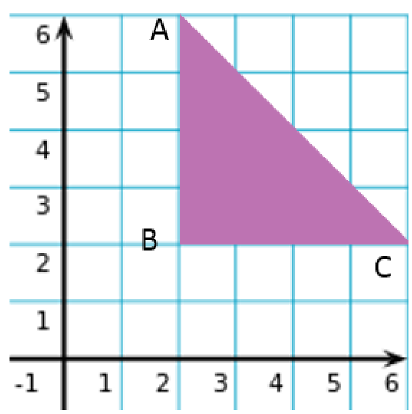
Is the new figure an enlargement or reduction of the original image?

Since the new figure is larger and our scale factor was greater than 1, the new image is an enlargement

Example 3

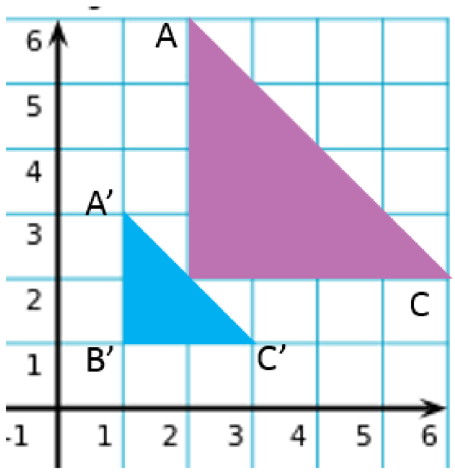
Given triangle ABC with coordinates A(2, 6), B(2, 2), C(6, 2). Dilate ABC by a scale factor of 1/2 with the origin as the center of dilation.

First, we graph our original triangle in the coordinate plane.



Multiply the coordinates of each point by the scale factor $\frac{1}{2}$ or simply divide each by 2. This gives A'(1, 3), B'(1, 1), C'(3, 1)

Plot the object and the image on the same graph.



Is the image an enlargement or reduction of our original figure?

Since the scale factor was a fraction less than 1 and the new triangle is smaller than the original, it is a reduction.

For all the cases we have done, the centre of dilation is the origin (0, 0).

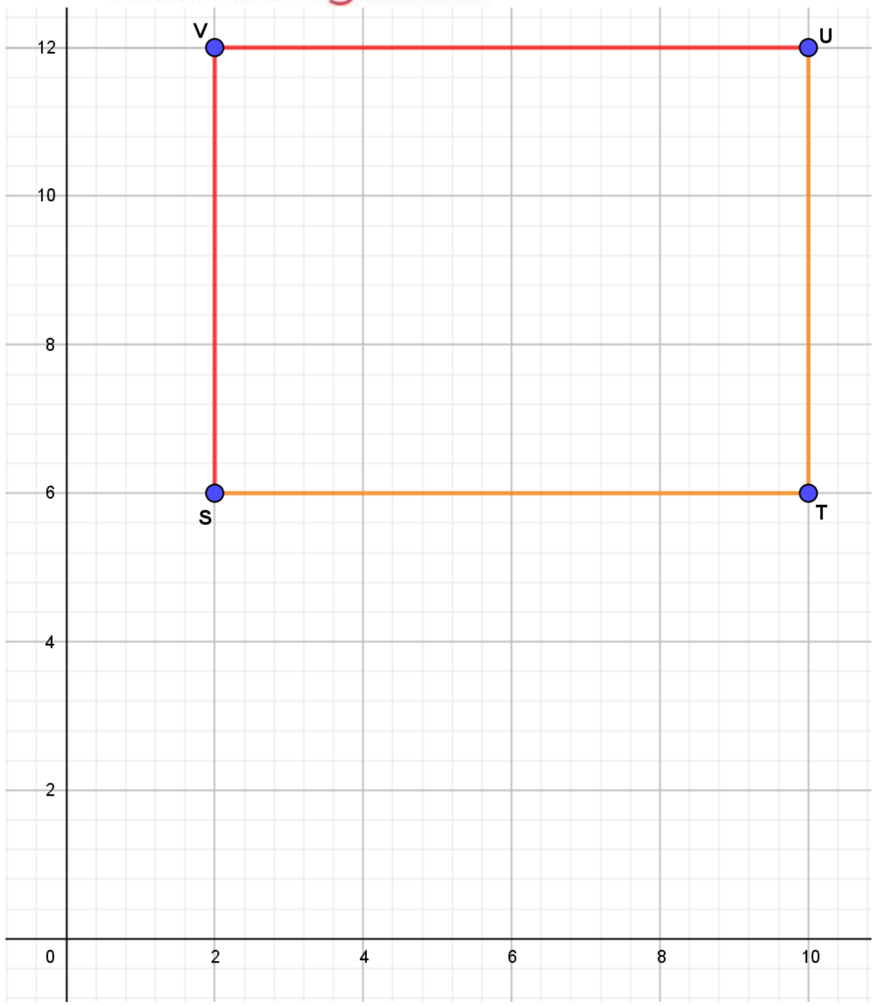
What happens when the centre is not the origin?

Example 1

Draw the rectangle with vertices S(2, 6); T(10, 6); U(10, 12) and V(2, 12).

Draw its image under a dilation with scale factor 0.5 and centre of dilation (2, 2).

Step 1



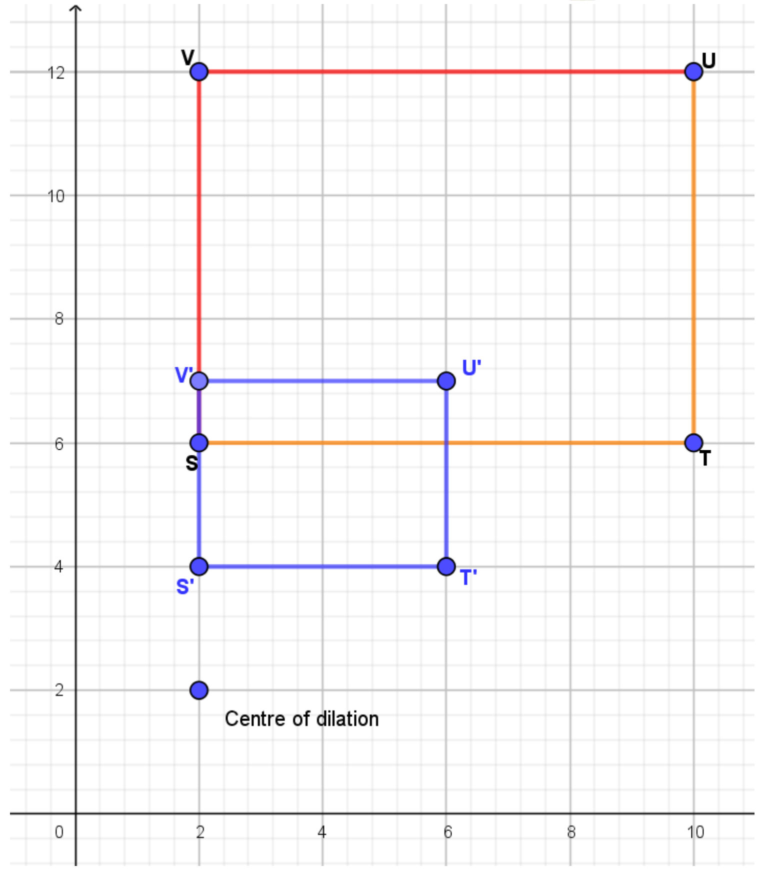
Step 2

Consider one of the points, say point **S**.

Determine the distance of point **S**, from the centre of dilation which is the point (2, 2)

The distance of S(2, 6) from the point (2, 2) is 4units. Since the Scale Factor is 0.5, multiply 4 by 0.5. i.e $4 \times 0.5 = 2$. It means the image **S'** will be 2 units from this centre (2, 2).

Determine the point that is 2units from the centre and is on the line connecting the center and point S. Plot this point. This is S’ the image of the point S. With one point plotted, the other points can easily be plotted to give the figure beside

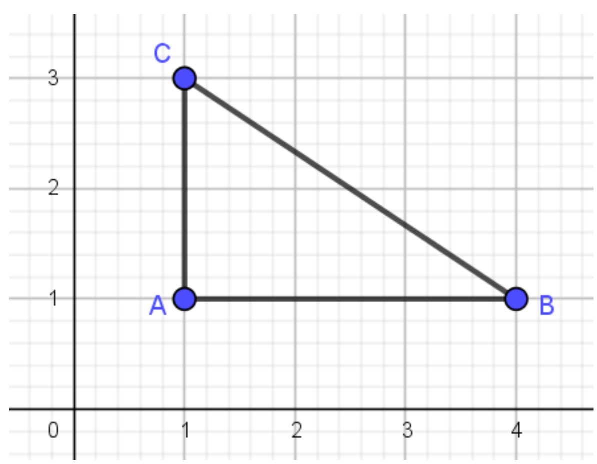


Example 2:

Consider triangle ABC with coordinates of vertices as A(1, 1); B(4, 1) and C(1, 3). Dilate this triangle by a scale factor of 2 centre (4, 2).

Step 1:

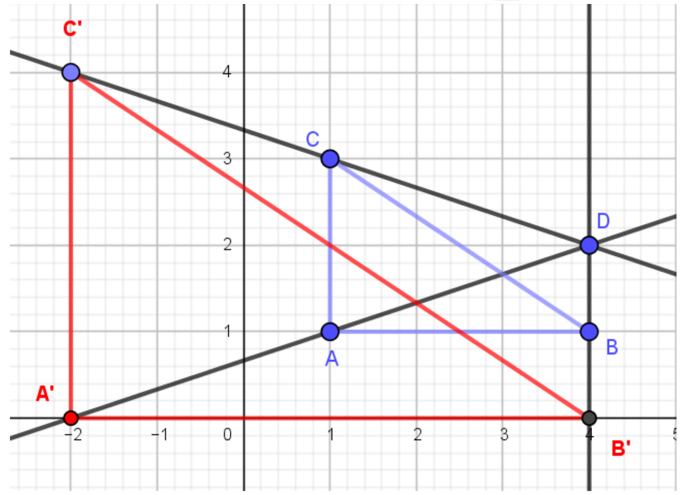
Plot the points A, B and C on a coordinate plane.



Step 2. Plot the centre of dilation D(4, 2)

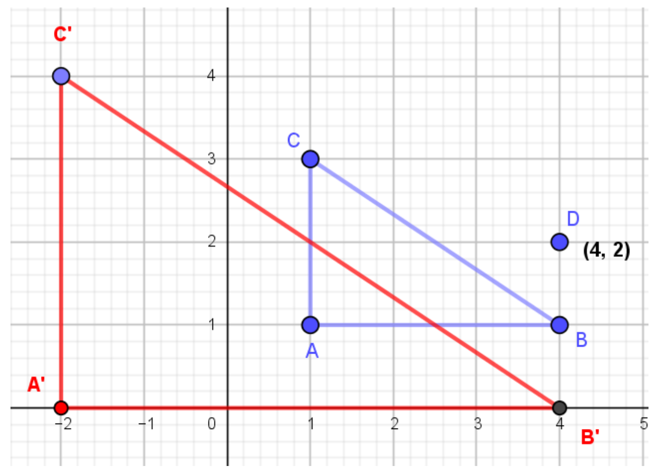
Step 3. Draw lines connecting each point and the centre of dilation.

Step 4. Using the relation between distance of object from centre of dilation and distance of image from dilation determine the image points.



Erasing the lines connecting the points their images and the centre of dilation gives the object and its image.

The coordinates of the image points are: A' (-2, 0); B'(4, 0) and C' (-2, 4)

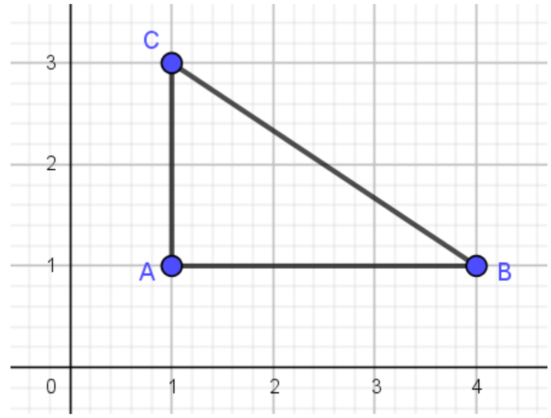


Examples 3:

Consider triangle ABC with vertices having coordinates A(1, 1); B(4, 1) and C(1, 3). Let it be dilated by scale factor 2 with centre (-1, 1) to triangle DEF such that the image of A is D, image of B is E and image of C is F. Find the coordinates of the image points.

Step 1

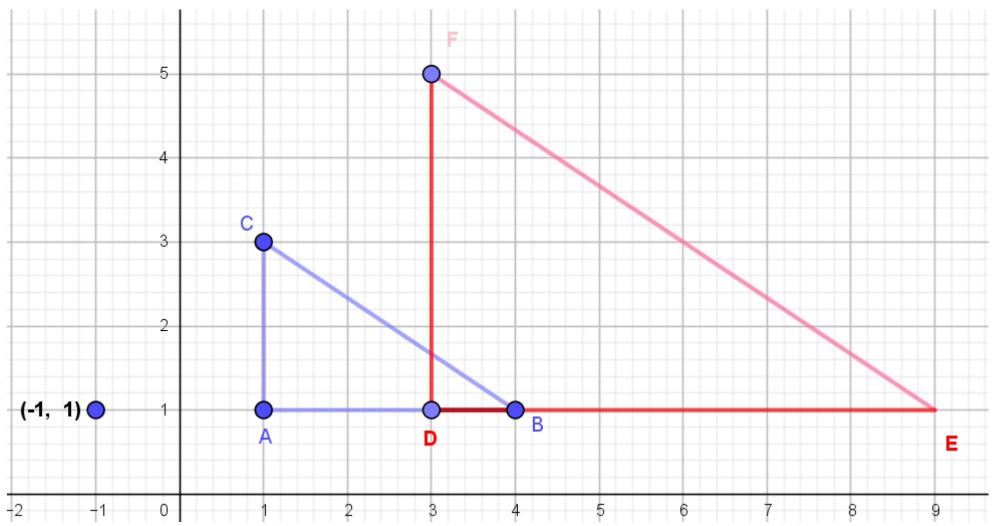
Plot the points A, B and C on a coordinate plane.



Step 2: Plot the centre of dilation.

Step 3. Draw lines connecting each point with the centre of dilation.

Step 3. Using the relationship between distance of object from centre of dilation and distance of its image, plot the image points and draw the figure



D(3, 1); E(9, 1) and F(3, 5)

Points to Remember

Dilation is when we enlarge or reduce a figure without changing its shape and orientation.

A description of a **dilation** includes the scale factor (or ratio) and the center of the **dilation**.

The **scale factor** refers to the change in size.

The **center of dilation** is a fixed point in the **plane** about which we are dilating the object.

If the scale factor is greater than 1, the image is an enlargement.

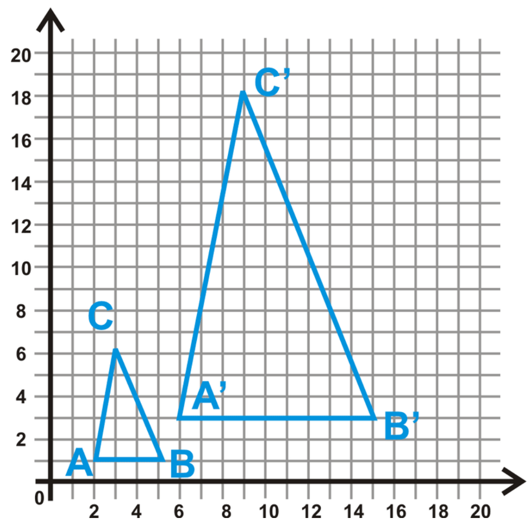
If the scale factor is less than 1, the image is a reduction.

The distance of the object and that of the image from the centre is the scale factor multiple of each other.

Usually, the center of dilation is the origin, (0,0). However, any point in the coordinate plane can be the centre of dilation.

Home work

1. This figure by the side represents a dilation. Triangle ABC has been dilated to triangle A’B’C’.
Determine the scale factor and the center of dilation.



2. Dilate triangle A’B’C’ by a scale factor $\frac{2}{3}$ and centre (0, 0). Give the coordinates of the image vertices, plot the points and draw the triangle on the same diagram.

Lesson 2: Translation

Objectives: At the end of this lesson you should be able to:

- Describe a translation;
- Identify situations of translation in real life;
- Perform or illustrate translation of plane figures on the coordinate system;

Rationale:

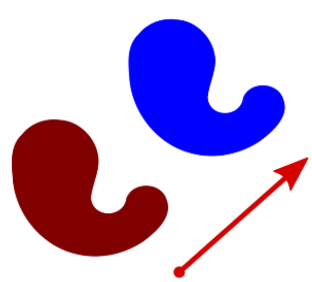
In our homes and at times in our classrooms, we move things around by changing their positions.

Didactic material:

Grid
Square paper or graph paper
Graph board or cardboard paper make with squares.

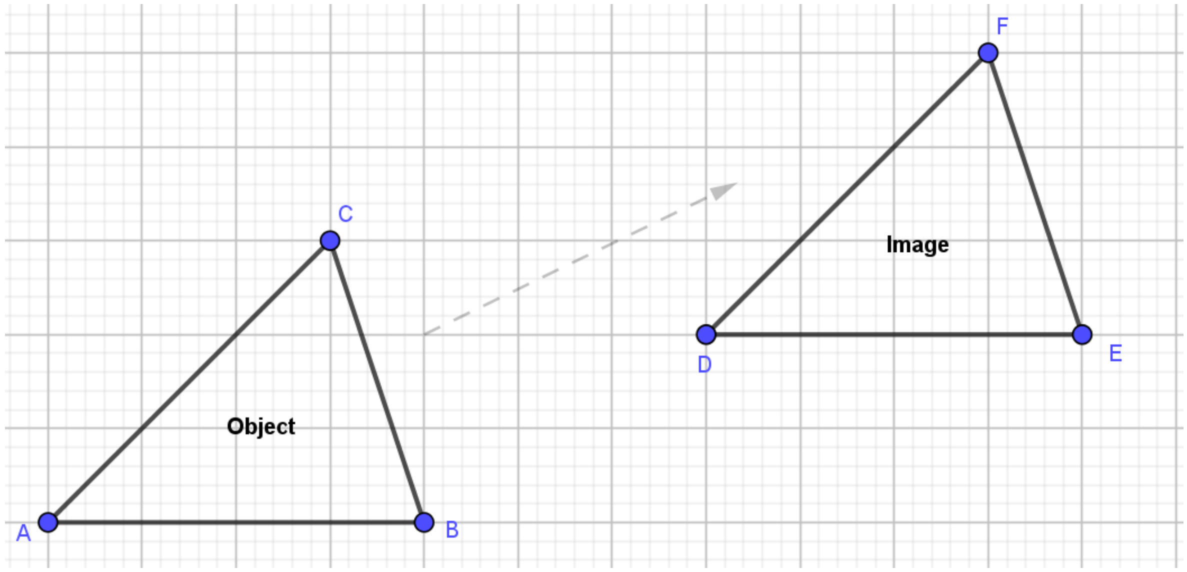
Introduction

Translation is moving an object in space without changing its size, shape or orientation. The object is slide in any direction without turning it (without changing its orientation).



The deep red item is the object or pre-image and the item in blue the image.

Consider this figure



In the above figure, triangle ABC has been transformed onto triangle DEF, such that the image of point A is D, the image of point B is E and the image of point C is F.

Activity

1. Consider the point A and its image point D. Starting from A, count the number of units to move to the right and number of units to move up to get to point D.
2. Do the same for each of the points and its corresponding image.
3. What can you conclude for any other point on the object?

Solution

1. Starting with point **A** move 7 units to the right then 2 units up to get to the image point **D**.
2. Starting with point **B** to get to its image **E**, move 7 units to the right then 2 units up. Similarly starting with point C, to get to its image F, move 7 units to the right then 2 units up.
3. From each point of the object, move 7 units to the right then 2 units up to get to the corresponding image point.

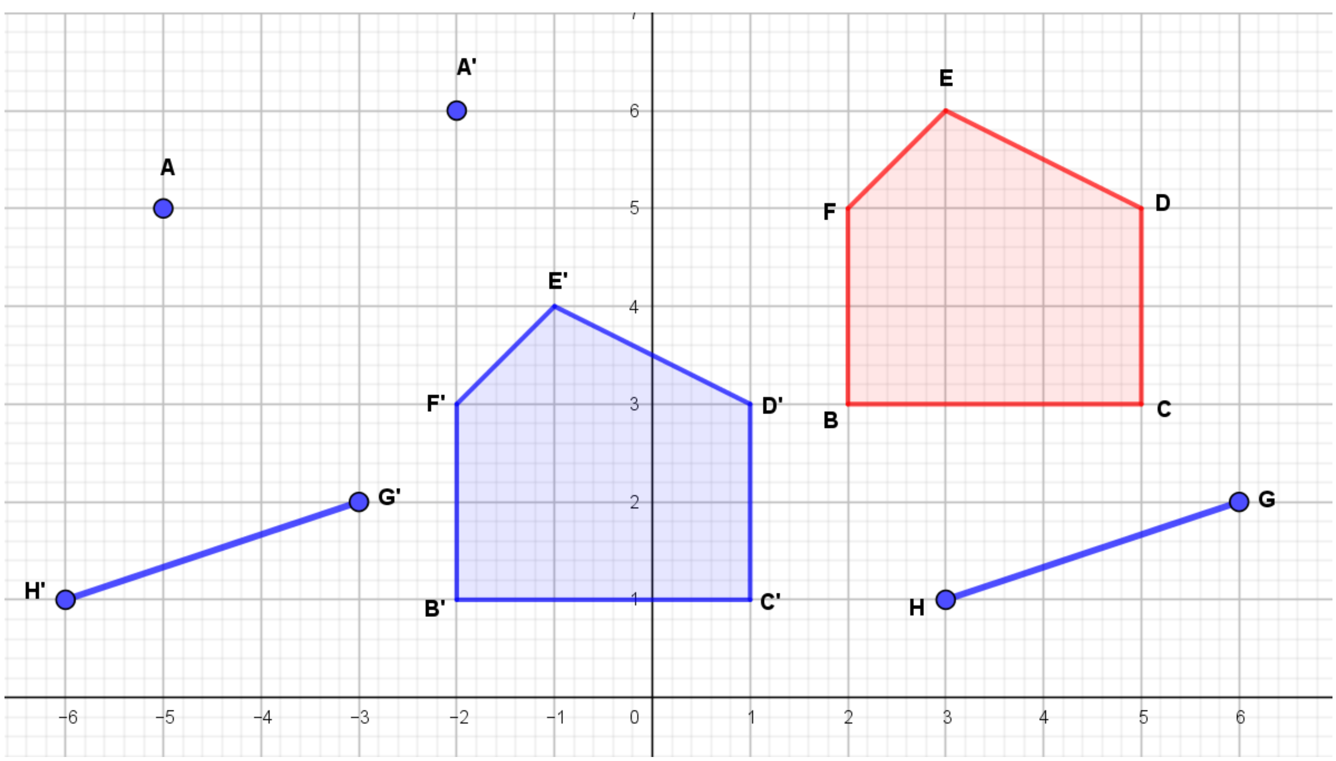
The Mathematical way to describe this translation is by the column vector $\binom{7}{2}$, and can also be written: (x, y) $\longrightarrow (x+7,\ \ y+2)$ because you have moved 7 spaces in the positive *x*-direction and 2 spaces in the positive *y*-direction.

Summary:

In a translation, every point of the object must be moved in the same direction and for the same distance. The object is not altered in any other way, only its position is changed.

Example 1:

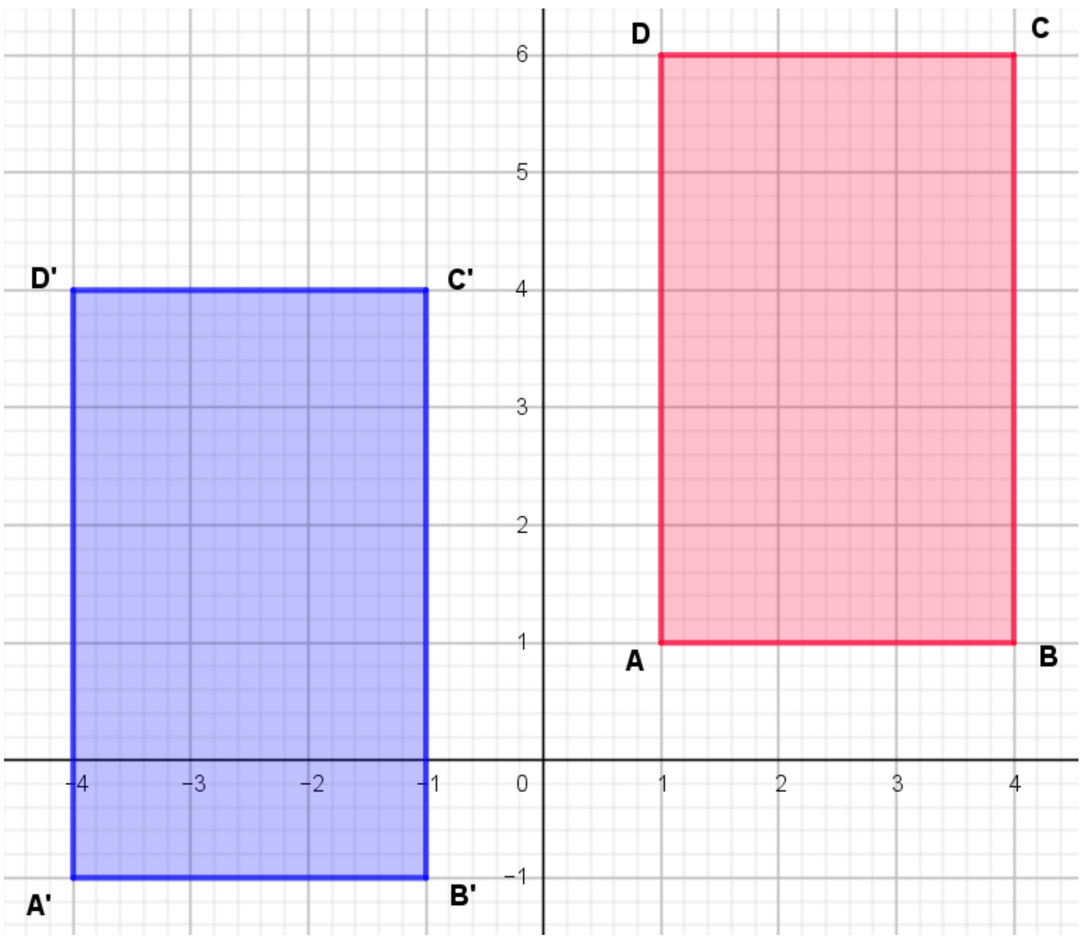
The coordinate plane with the figures below, will be used for the examples.



1. The point A has moved 3 places to the right and 1 place upward so it has been translated to the image point A' by the column vector $\binom{3}{1}$. This can also be written (x, y) $\longrightarrow$ (x +3, y +1). Movement to the right is positive, movement to the left is negative, movement upward in positive and movement downward in negative.

2. The line HG has moved 9 spaces to the left and no spaces downward or upward so it has been translated to image H'G' by the column vector $\binom{-9}{0}$. This can be written as (x, y) ⟶ (x - 9, y + 0).
3. The figure in red has been translated to figure in blue by the column vector $\binom{-4}{-2}$, also written as (x, y) ⟶ (x - 4, y – 2).
4. The polygon in blue can be translated to the polygon in red by the translation vector $\binom{4}{2}$. Written as (x, y) ⟶ (x + 4, y + 2).

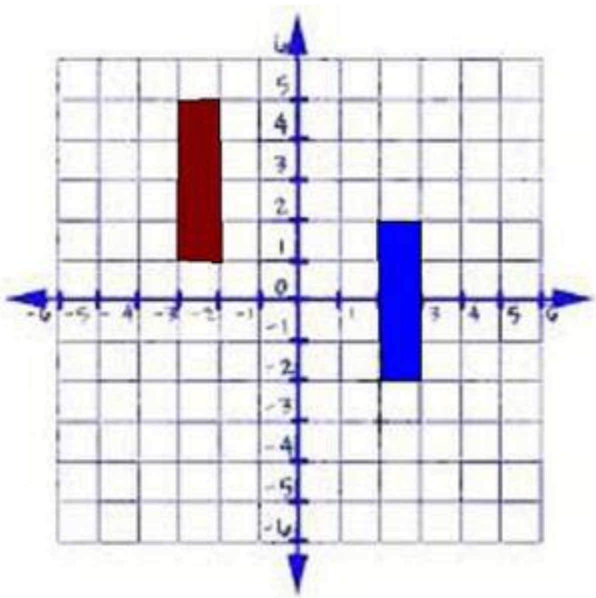
Example 2:



Write out the translation that displaces rectangle ABCD (in red) to rectangle A'B'C'D' (in blue).
What is the translation that will take A'B'C'D back to ABCD?
Each point on rectangle ABCD has moved 5 places to the left then 2 places downward.

The column vector is $\binom{-5}{-2}$, which could equally be written as (x, y) → $(x-5, y-2)$.
To translate rectangle A'B'C'D' to rectangle ABCD, move 5 places to the right then 2 places upward. This gives the column vector $\binom{5}{2}$, which is also written as (x, y) → $(x+5, y+2)$.

Example 3:



From rectangle in blue, move each point to the left five spaces and up three spaces. This gives the image rectangle in dark red.

Points to remember:

In geometry, translation simply means Moving, without rotating, without resizing or anything else.

To translate a shape, every point of the shape moves:

- ✓ The same distance
- ✓ The same direction (to the right, to the left, upward or downward)
- ✓ Movement to the right is positive, movement to the left is negative, movement upward in positive and movement downward in negative.

The translation vector can be represented by a column vector that indicates the number of places to move either to the left or to the right and upward or downward.

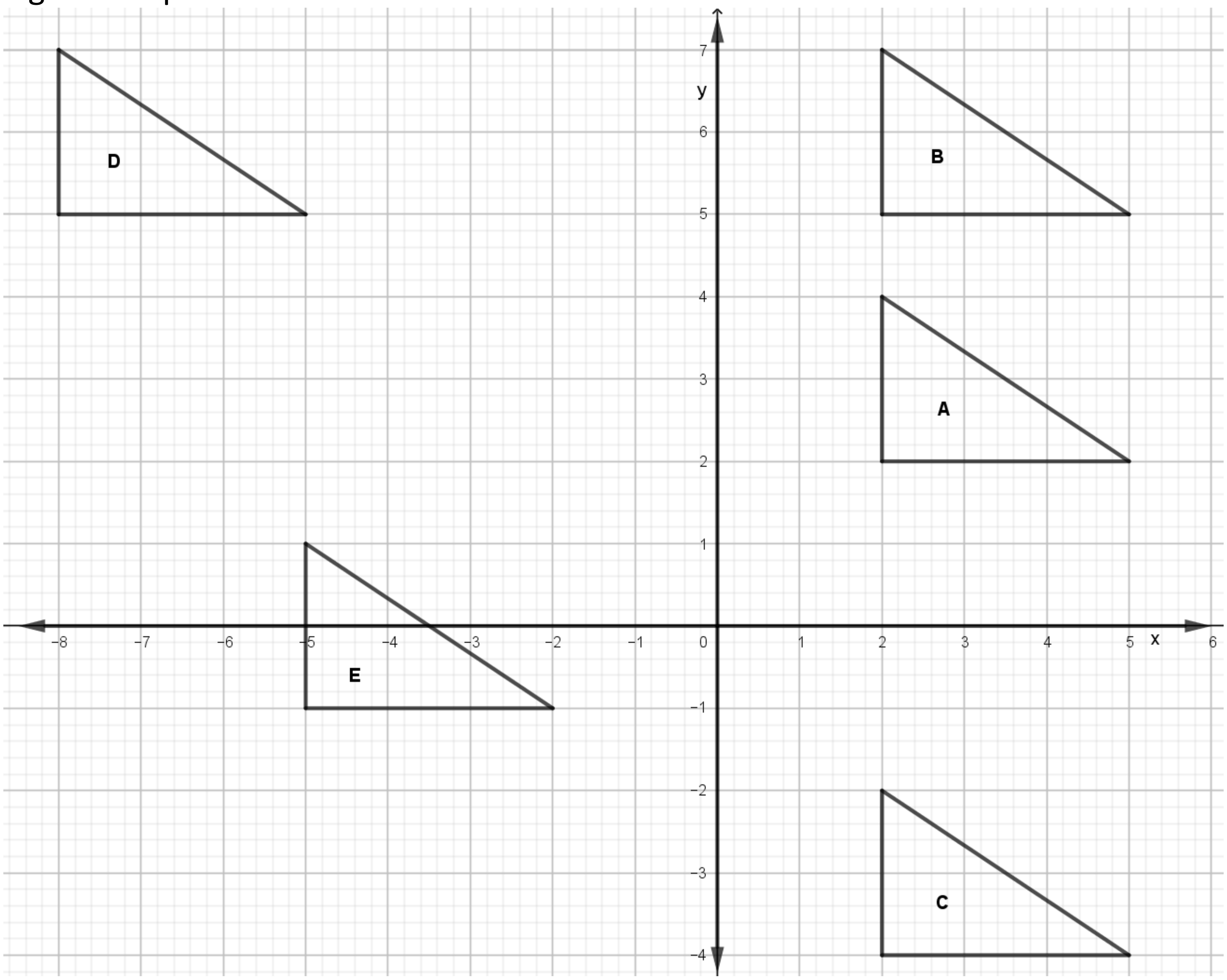
It is usually represented as $(x, y) \longrightarrow (x + k,\ y + p)$, where (x, y) are the coordinates of the object, k the number of spaces to move either to the right or to the left and p the number of places to move upward or downward.

Homework:

1. Using the Cartesian plane below with the plane figures A, B, C, D and E, find the column vector for each of the following translation.

   a) A onto C; b) B onto C; c) B onto C; d) D onto E; e) E onto D; f) A onto E.

Figure for question 1



1. a) Draw the object triangle ABC at A(-4, -1); B(-4, 1); C(-1, -1) and shade it.

   a) Draw the image of triangle ABC under the translations describe by the vectors below. For each, write down the coordinates of each image point.

i) $\binom{3}{2}$; ii) $\binom{-3}{-2}$; iii) $\binom{-3}{2}$; iv) $\binom{3}{-2}$

The next lesson will be on Reflection