SAMPLE LESSON: MATHEMATICS

Class: Form 3

Title of Module: *Algebra and Logic*	**Title of Chapter:** Algebraic Processes
Title of Lesson: Solving Quadratic Equations by completing the square	**Duration of Lesson: 55mins**

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School: **AIMS-TTP COP**
Class : FORM 4
No. on Roll: **Boys:** **Girls:**

Sequence: 3
Duration: 50 Minutes
Average age: 16

Module: ***Algebra and Logic***
Topic: ***Algebraic Processes***
Lesson: ***Solving Quadratic Equations by completing the square***
Objectives: At the end of this lesson, students should be able to:

1) Complete the square for a quadratic expression in one variable
2) Solve quadratic equations using the method of completing the square
3) Apply this concept in daily life like in the construction of a house, bridge, etc.

Pre-requisite knowledge: Students are able to:

- ✓ Identify a quadratic expression in one variable
- ✓ Identify a perfect square
- ✓ Carry out basic operations on (real) numbers and variables
- ✓ Expand brackets
- ✓ Find areas of squares and rectangles

Motivation: Students often face a great difficulty in memorizing the traditional procedure (*add and subtract the square of half of the coefficient of* x) usually given for completing the square. Thus, we intend to bring in this lesson a geometrical hands-on approach to alleviate this challenge using familiar mathematical manipulations.

Didactic materials: Chalk (white and colored), Duster, cardboard papers of different colors and sizes.
REFERENCES: - Mathematics Teaching Syllabus for Forms 3, 4 and 5; Ministry of Secondary Education: Cameroon.
- Andrew T. Tamambang et al., (2017), Interactions in Mathematics, Students' Book – O' Level; South Africa: Cambridge University Press.

- https://brilliant.org/wiki/completing-the-square/

- https://medium.com/i-math/how-to-complete-the-square/

STAGES/ DURATION	TEACHING/LEARNING activities	TEACHER'S ACTIVITIES	LEARNERS' ACTIVITIES	LEARNING POINTS	observations
Introduction (**7 Mins)**	A/-**Verification of prerequisite knowledge** 1. Which of the following is/are a quadratic expression(s)? a) $2x + 5$ b) $x^2 + 4x$ c) $3x^2 - 12x + 2.5$ d) $x^2 + y + 5x - 7$ 2. What is a perfect square? Give examples. 3. What is the product of $(x + 2)$ and $(x + 3)$? 4. A rectangle has length x units and width 5 units. What is its area?	-Prepares the chalkboard -Asks the entry questions, writing on the chalkboard when need arises. -Listens to students' response and comments appropriately. -Points out to particular students to answer questions	Indicate by show of hand to answer the questions asked Work out the responses.	Knowledge of perfect squares **<u>Expected Responses</u>**: 1. b) and c) 2. A number obtained by squaring a whole number is a perfect square (Or the value obtained by finding the area of a square). For example: 16, 36, x^2. 3. $(x + 2)(x + 3) = x^2 + 5x + 6$ 4. $5x$	
	/-**Problem situation** **Mrs. Jones is tiling his parlor with 50cm×50cm tiles The floor is of dimension $8m \times 8m$. He has done the work and the tiles he bought got finished and he is left with a 4m² square portion to complete the work. How many tiles still of**	Give out copies of the Problem situation if typed out, or paste copies on the wall. Ask one of the students to read out the problem situation Listen to students' responses and appreciate.	Read problem situation. Reflect on what Mr. Jones can do to know the exact number of tiles to buy to complete the work Brainstorm to solve the problem.		

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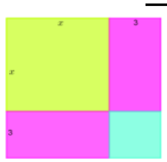
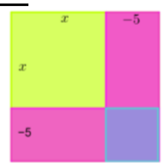
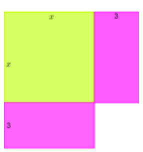
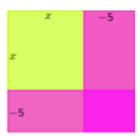
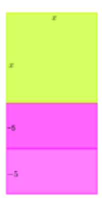
	50cm×50cm will he need to complete the work?	Introduces and writes lesson title on the chalkboard.	Give their solutions.		
Lesson Development (30mins)	<u>Activity 1</u>   Each group is given a set of four (04) paper cut outs alongside an A4 paper to write answers on: Task 1-1: Puzzle and form a square (like the ones above). Task 1-2: Find and write the total area of the square in *two different ways.* <u>Activity 2</u>    	Puts students into three groups, ensuring a balance in the level of understanding of the learners and genders. Appoints group leaders Shares the "*Algebra kits*" to the different groups. Gives instructions for each task under each activity. Moves around the classroom, giving hints, ensuring the students are following the instructions correctly.	Move and arrange themselves in their different groups. Receive the "*Algebra kits*" Listen carefully to the instructions as the facilitator gives. Work as a team to bring out the expected results. Group leaders coordinating.	1. **<u>Completing the Square</u>** Completing the square is a technique of manipulating a quadratic expression into a perfect square plus or minus a constant. This technique is commonly used in solving quadratic equations, determining maximum and minimum values of quadratic functions and in deriving the quadratic formula. Recall that: $(x+3)^2 = x^2+6x+9$ (1) $(x+4)^2 = x^2+8x+16$ (2) $(x-5)^2 = x^2-10x+25$ (3) In general, $(x+a)^2 = x^2+2ax+a^2$ $(x-a)^2 = x^2+(-2a)x+(-a)^2$ $= x^2-2ax+a^2$ From (1), $x^2+6x = (x+3)^2-9$ $= \left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2$ From (2), $x^2+8x = (x+4)^2-16$ $= \left(x+\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2$	Compare results from Activity 1 Multiplying two binomials 9 is subtracted from both sides, we observe that 3 is half of 6 – the coefficient of x.

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	Task 2-1: Remove the green portion and write the area of the remaining portion in *two different ways.* Task 2-2: Rearrange the remaining three paper cut outs to form a rectangle. Write down the area of the rectangle formed. Task 2-3: Compare your results from task 2-1 and task 2-2. (N.B. For Activity 1, the different areas are calculated by adding the areas of the four paper cut out or considering the total length and width and then multiplying For Activity 2, it is the area of the supposed-complete square minus the area of the removed green paper cut out.)	Encourages teamwork Supervises the work in the groups. Takes down the results from each group. Make necessary corrections. Dictates notes, while writing on the chalkboard when need be.	Agree on their findings before writing. A representative from each group presents their findings Follow up with the teacher as he comments on the results. Listen and take notes into their notebooks.	From (3), $x^2 - 10x = (x-5)^2 - 25$ $= \left(x - \frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2$ Thus, we deduce that: $x^2 + kx = \left(x + \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$ $x^2 - kx = \left(x - \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$ Examples: $i)\ x^2 + 18x = (x+9)^2 - 81$ $ii)\ x^2 - 5x = \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2$ $iii)$ In our opening problem, Mr. Jones has tiled 60m^2 of the floor. The floor needs to be tiled in a *perfectly squared manner*, so he has to complete the square with 16 tiles which is equivalent to an area of 4m^2. 2. **Solving Quadratic Equations by Completing the Square** Here, we complete the square for the quadratic expression and then equate to zero, the solving follows suit.	Likewise for the others. Compare results from Activity 2

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Exercises of Application (7mins)	1. Complete the square for: a) $x^2+12x+20$ b) x^2-4x-5 2. Solve, by completing the square, the quadratic equation: a) $x^2+12x+20=0$ b) $x^2-7x+4=0$	-Gives learners time to attempt solving the questions (some on the chalkboard) while he moves around giving hints. -Asks learners to give their answers. -Does the correction of the questions on the chalkboard, while asking questions, explaining and making reference to the results of the activities. Checks/clarifies to be sure students are understanding. Moves round the class to ensure students are correctly taking the solutions into their notebooks.	Solving the questions Give their attempted solutions. Follow up the correction with the facilitator, giving verbal responses to any questions being asked. Ask questions for clarity. Copy solutions into their notebooks.	**<u>Solution</u>** 1. a) $x^2+12x+20=(x+6)^2-36+20$ $=(x+6)^2-16$ b) $x^2-4x-5=(x-2)^2-4-5$ $=(x-2)^2-9$ 2. a) $x^2+12x+20=0$ $(x+6)^2-36+20=0$ $(x+6)^2-16=0$ $(x+6)^2=16$ $x+6=\pm\sqrt{16}$ $x=-6\pm 4$ $\therefore \boxed{x=-10,-2}$ b) $x^2-7x+4=0$ $\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+4=0$ $\left(x-\frac{7}{2}\right)^2-\frac{33}{4}=0$ $\left(x-\frac{7}{2}\right)^2=\frac{33}{4}$ $x-\frac{7}{2}=\pm\sqrt{\frac{33}{4}}$ $\therefore \boxed{x=\frac{7}{2}-\frac{\sqrt{33}}{2}, x=\frac{7}{2}+\frac{\sqrt{33}}{2}}$	

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		Fills the logbook (Record of work)			
Conclusion (6mins)	**Assignment** 1. Complete the square for each of the following quadratic expression: a) x^2+6x b) $x^2-18x+31$ c) x^2+4x+4 d) $3x^2+15x-12$ 2. Use the technique of completing the square to solve the quadratic equation: a) $x^2-3x+2=0$ b) $x^2-2x-2=0$, giving your answer in exact form. c) $2x^2+4x=1$, giving your answer to two decimal places. 3. A rectangular piece of land is such that its length is three more than its width. a) Let w be the width and l the length. Write down an	Dictates questions clearly, writing on the chalkboard whenever need arises.	Listen and take questions in their notebooks.	**Correction of Assignment** 1. a) $x^2+6x=(x+3)^2-9$ b) $x^2-18x+31=(x-9)^2-81+31$ $=(x-9)^2-50$ c)) $x^2+4x+4=(x+2)^2-4+4$ $=(x+2)^2$ d) $3x^2+15x-12=3[x^2+5x-4]$ $=3\left[\left(x+\frac{5}{2}\right)^2-\frac{25}{4}-4\right]$ $=3\left[\left(x+\frac{5}{2}\right)^2-\frac{41}{4}\right]$ $=3\left(x+\frac{5}{2}\right)^2-\frac{123}{4}$ 2. a) $x^2-3x+2=0$ $\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+2=0 \Longrightarrow \left(x-\frac{3}{2}\right)^2=\frac{1}{4}$ $x-\frac{3}{2}=\pm\frac{1}{2}$ $\therefore \boxed{x=1,2}$ b) $x^2-2x-2=0$ $(x-1)^2-1-2=0$ $(x-1)^2=3$	Factoring out 3 Multiplying through by 3

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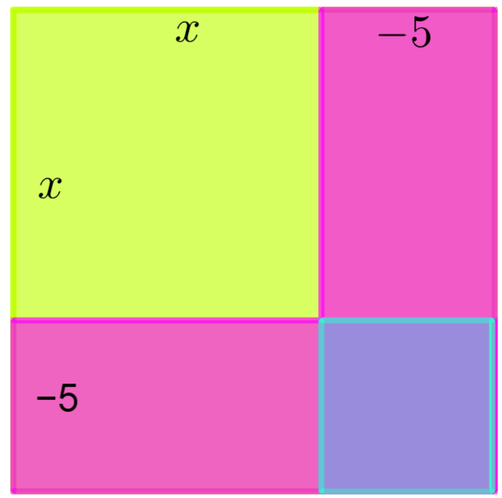
	expression for the length in terms of the width. b) Write an expression for its area, A, in terms of w c) Given that the area is 4 square units, show that $w^2+3w-4=0$ d) Hence, by completing the square, find the two values of w. Which of these two values is contextually correct? (N.B. In this lesson, only quadratic expressions with leading coefficient 1 have been treated. However, the assignment has cases with the leading coefficient not 1. This is intended to raise the learners' curiosity and keep them thinking. The approach will be better explained in the next learning session.)			$x-1=\pm\sqrt{3}$ $\therefore \boxed{x=1-\sqrt{3}, 1+\sqrt{3}}$ (Exact form) c) $2x^2+4x=1 \Longrightarrow 2x^2+4x-1=0$ $2\left[x^2+2x-\frac{1}{2}\right]=0$ $2\left[(x+1)^2-1-\frac{1}{2}\right]=0$ $2\left[(x+1)^2-\frac{3}{2}\right]=0 \Longrightarrow (x+1)^2-\frac{3}{2}=0$ $\Longrightarrow x+1=\pm\frac{3}{2} \Longrightarrow x=-1\pm\frac{3}{2}$ $\therefore \boxed{x=-2.50, 0.50}$ 3. a) $l=w+3$ b) $A=lw=(w+3)w=w^2+3w$ c) $A=4 \Rightarrow w^2+3w=4$ $\Rightarrow w^2+3w-4=0$ d) $w^2+3w-4=0$ $\Longrightarrow \left(w+\frac{3}{2}\right)^2-\frac{9}{4}-4=0$ $\left(w+\frac{3}{2}\right)^2=\frac{25}{4}$ $w=-\frac{3}{2}\pm\frac{5}{2}$ $w=-4, 1$ The value $w=1$ is contextually correct because lengths are never negative.	 Factoring out 2 Dividing all through by 2

<u>Activity 1</u>

Each group is given a set of four (04) paper cut outs alongside an A4 paper to write answers on:

Task 1-1: Puzzle and form a square (like the ones above).

Task 1-2: Find and write the total area of the square in *two different ways.*

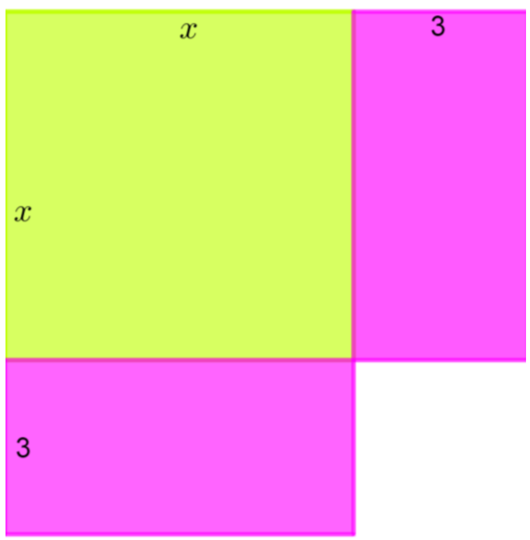
 $X^2 + 6x + 9$	 $X^2 - 10x + 25$

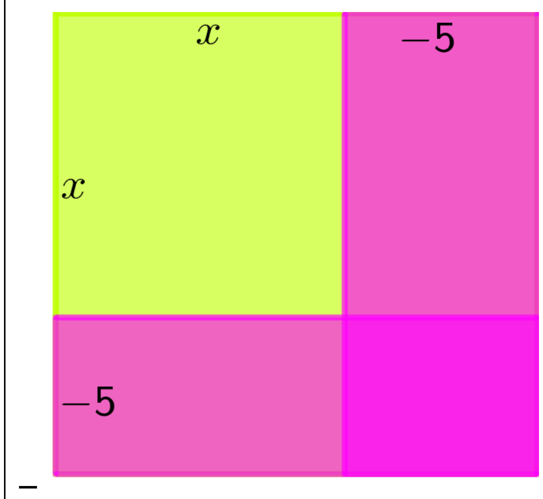
<u>Activity 2</u>

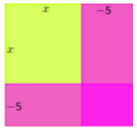
Task 2-1: Remove the green portion from the figure formed in activity 1 and write the area of the remaining portion in *two different ways.*

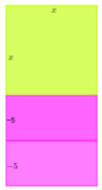
Task 2-2: Rearrange the remaining three paper cut outs to form a rectangle. Write down the area of the rectangle formed.

Task 2-3: Compare your results from task 2-1 and task 2-2.









(N.B. For Activity 1, the different areas are calculated by adding the areas of the four paper cut out or considering the total length and width and then multiplying

For Activity 2, it is the area of the supposed-complete square minus the area of the removed green paper cut out.)