

SAMPLE LESSON: MATHEMATICS

Class: Upper Sixth Mathematics

Module: Plane Geometry and Solid Figures	TOPIC: Complex Numbers
Title of Lesson: Polar form (trigonometric form) and exponential form of a non-zero complex number	Duration of Lesson: 120mins

Name of Authors: Inspectorate of Pedagogy/Sciences for the Far North Region



Module 4 : PLANE GEOMETRY AND SOLID FIGURES

Topic: Complex Numbers

Lesson: Polar form (trigonometric form) and exponential form of a non-zero complex number

Objectives: At the end of this lesson, the learners should be able to:

- 1) Calculate the modulus of a complex number
- 2) Find Argument of a non-zero complex number;
- 3) Give the polar form of a non-zero complex number;
- 4) Give the exponential form of a non-zero complex number.

Key question:

What is the polar form of a non-zero complex number?

Prerequisite knowledge:

- \checkmark Verify whether students can calculate the norm of a vector.
- ✓ Verify whether students can determine the measurement of the angle between the positive X-axis and a vector.

Motivation: The study of complex numbers comes to reinforce our knowledge and skills necessary to study plane geometry.

Didactic materials

Chalk, colour chalk, Chalkboard, ruler and set square.

REFERENCES

- EWANE ROLAND ALUNGE. Advanced Level Pure Mathematics Made Easy First Edition. -Pure Mathematics With Mechanics Teaching Syllabuses (January 2020)







Stages /	Teaching /Learning activities	Teacher's	Learners'	Learning Points	Observations
Duration		Activities	Activities		
Introduction	Verification of Pre-requisites			Verification of Pre-requisites	
(5mins)	Exercise a) Calculate the norm of $\overrightarrow{OA}\begin{pmatrix}0\\4\end{pmatrix}$ given in the Cartesian plane. b) Determine the measure of the angle between \overrightarrow{i} and \overrightarrow{OA} .	-Copies questions on the board -Calls students to the board	-Solve on the board as called by the teacher	Exercise a) Calculate the norm of $\overrightarrow{OA}\begin{pmatrix}0\\4\end{pmatrix}$ given in the Cartesian plane. b) Determine the measure of the angle between $\vec{\iota}$ and \overrightarrow{OA} .	
Lesson	<u>Activity</u>			Activity	
Development	In an orthonormal reference system (O, <i>ī</i> , <i>j</i>),we	-Copies	-Follow the	In an orthonormal reference system (O, $\vec{\iota}$,	
and Summary	consider the points A,B and C with respective	activity on	instruction	\vec{j}),we consider the points A,B and C with	
(100mins)	affixes $z_A = 2$, $z_B = 3i$ and $z_C = 2 + 2i$ 1) Plot the points A, B and C in this reference system. 2) Calculate the values of OA,OB and OC. 3) Determine the measure of the following angles: a) The angle between \vec{i} and \overrightarrow{OA} . b) The angle between \vec{i} and \overrightarrow{OB} . c) The angle between \vec{i} and \overrightarrow{OC} .	the chalkboard -Instructs students to copy in their notebooks and allows them 10 minutes to research	s and carry out the activity while interacting with each other	respective affixes $z_A = 2$, $z_B = 3i$ and $z_C = 2 + 2i$ 1) Plot the points A, B and C in this reference system. 2) Calculate the values of OA,OB and OC. 3) Determine the measure of the following angles: a) The angle between \vec{i} and \overrightarrow{OA} . b) The angle between \vec{i} and \overrightarrow{OB} . c) The angle between \vec{i} and \overrightarrow{OC} .	







Stages /	Teaching /Learning activities	Teacher's	Learners'	Learning Points	Observations
Duration		Activities	Activities		
	Resolution		-Сору	<u>Resolution</u>	
	1)		notes in	1)	
			their books		
	$\alpha = 45^{\circ}$ $2)OA = \sqrt{0^{2} + 2^{2}} = 2,OB\sqrt{3^{2} + 0^{2}} = 3$ $OC = \sqrt{2^{2} + 2^{2}} = \sqrt{8}$	-Copies notes on the board -Explains concepts	Copy notes in their notebooks	$\alpha = 45^{\circ}$ $\alpha =$	
	3)According to the figure above, a)The angle between $\vec{\imath}$ and \overrightarrow{OA} is $\frac{\pi}{2}$ b)The angle between $\vec{\imath}$ and \overrightarrow{OB} is 0 c) The angle between $\vec{\imath}$ and \overrightarrow{OC} is $\frac{\pi}{4}$. Observation For example	-Dictates notes	Follow up	3)According to the figure above, a)The angle between \vec{i} and \overrightarrow{OA} is $\frac{\pi}{2}$ b)The angle between \vec{i} and \overrightarrow{OB} is 0 c) The angle between \vec{i} and \overrightarrow{OC} is $\frac{\pi}{4}$. <u>Observation</u> For example	







Stages /	Teaching /Learning activities	Teacher's	Learners'	Learning Points	Observations	
Duration		Activities	Activities			
	\rightarrow) $\sqrt{8}$ Is called modulus of the complex number		as the	\rightarrow) $\sqrt{8}$ Is called modulus of the complex		
	z = 2 + 2i.		teacher	number $z = 2 + 2i$.		
	\rightarrow) (\vec{i} ; \overrightarrow{OC}) = $\frac{\pi}{4}$ will be called argument of the		explains,	\rightarrow) (\vec{i} ; \overrightarrow{OC}) = $\frac{\pi}{4}$ will be called argument of the		
	complex number $z = 2 + 2i$.	asking and	complex number $z = 2 + 2i$.			
	<u>Definition (</u> modulus)		questions	Definition (modulus) Let $z = a + bi$ be a complex number,		
	Let $z = a + bi$ be a complex number,	Solves the				
	We call the modulus of <i>z</i> the strictly positive real	examples	necessary	We call the modulus of <i>z</i> the strictly positive		
	number denoted by $ z $ and define by	one after	,	real number denoted by $ z $ and define by		
	$ z = \sqrt{a^2 + b^2}$ the other Example while	the other		$ z = \sqrt{a^2 + b^2}$		
			Example			
	Calculate the modulus of the complex number	questioning		Calculate the modulus of the complex number		
	z = 2 + 3i	the		z = 2 + 3i		
	$\underline{\text{Resolution}} z = \sqrt{2^2 + 3^2} = \sqrt{13}$	students	students		$\underline{\text{Resolution}} z = \sqrt{2^2 + 3^2} = \sqrt{13}$	
	Properties		<u>Properties</u>			
	Let z and z' two complex numbers			Let z and z' two complex numbers		
	\rightarrow) z =0 if and only if $z = 0$.			\rightarrow) z =0 if and only if $z = 0$.		
	$\rightarrow) z \times z' = z \times z' $			$\rightarrow) z \times z' = z \times z' $		
	$\rightarrow) z^n = z ^n \forall n \in \mathbb{N}$			$\rightarrow) z^n = z ^n \forall n \in \mathbb{N}$		
			\rightarrow) $ \bar{z} = z $			
			$\rightarrow) \left \frac{z}{z'} \right = \frac{ z }{ z' } \forall z' \in \mathbb{C}^*$			
				<u>Remark</u>		
			For any $M(z)$ and $M'(z')$			
	$MM' = Z_{M'} - Z_M $			$MM' = Z_{M'} - Z_M $		
	Definition (argument)			Definition (argument)		







Stages / Duration	Teaching /Learning activities	Teacher's Activities	Learners' Activities	Learning Points	Observations
	Let z be a non-zero complex number,			Let z be a non-zero complex number,	
	M the image of the complex number z ,in the			M the image of the complex number z , in the	
	complex plane with an orthonormal reference			complex plane with an orthonormal	
	system (O, <i>ī</i> , <i>j</i>),			reference system (O, ī, j),	
	-We call Argument of <i>z</i> the angle			-We call Argument of <i>z</i> the angle	
	$\left(\widehat{\vec{l}; OM}\right)$ denoted $Arg(z)$			$\left(\widehat{\vec{l}; OM}\right)$ denoted $Arg(z)$	
	- We call argument of z the measure of angle			- We call argument of <i>z</i> the measure of angle	
	$\left(\vec{l}; \overrightarrow{OM}\right)$ denoted $arg(z)$			$\left(\vec{l}; \overrightarrow{OM}\right)$ denoted $arg(z)$	
	$\operatorname{Arg}(z) = \left(\widehat{\vec{\iota}; OM}\right)$			$\operatorname{Arg}(\mathbf{z}) = \left(\widehat{\vec{\iota}; OM}\right)$	
	$\arg(z) = \operatorname{mes}(\widetilde{i; OM})$			$\arg(z) = \operatorname{mes}(\overline{i; OM})$	
	Example			Example	
	$\operatorname{Arg}(2+2i) = \left(\overline{\frac{\pi}{4}}\right)$			$\operatorname{Arg}(2+2i) = \left(\frac{\widehat{\pi}}{4}\right)$	
	$\arg(2+2i)=\frac{\pi}{4}$			$\arg(2+2i)=\frac{\pi}{4}$	
				Remark	
	<u>Remark</u>			If the affix of the vector $ec{u}$ is $z_{ec{u}}$,then	
	If the affix of the vector $ec{u}$ is $z_{ec{u}}$,then			$\arg(z_{\vec{i}}) = \max(\widehat{\vec{i};\vec{u}})$	
	$\arg(z_{\vec{u}}) = \max(\widehat{i; \vec{u}})$				
				Polar form of a non-zero complex	
	Polar form of a non-zero complex number			number	
	Let $z = a + ib$ be a non-zero complex number			Let $z = a + ib$ be a non-zero complex number	







Stages / Duration	Teaching /Learning activities	Teacher's Activities	Learners' Activities	Learning Points	Observations
	And M the image of <i>z</i> in an Argand Diagram.			And M the image of z in an Argand Diagram.	
	We suppose that $\arg(z) = \alpha$, $ z = r$			We suppose that $\arg(z) = \alpha$, $ z = r$	
				$b \qquad \qquad$	
	$\cos \alpha = \frac{a}{r} \rightarrow a = r \cos \alpha$			$\sin \alpha = \frac{1}{r} \rightarrow b = r \sin \alpha$	
	$\sin \alpha = \frac{b}{r} \rightarrow b = r \sin \alpha$			$z = r \cos \alpha + i r \sin \alpha = r(\cos \alpha + i \sin \alpha)$	
	$z = r \cos\alpha + ir \sin\alpha = r(\cos\alpha + i\sin\alpha)$			$z=r(\cos \alpha + i\sin \alpha)$ is called the polar	
	$z=r(\cos \alpha + i \sin \alpha)$ is called the polar			<i>z</i> .	







Stages /	Teaching /Learning activities	Teacher's	Learners'	Learning Points	Observations
Duration		Activities	Activities		
	(trigonometric) form of the complex number z .			This polar form can also be written as follow	
	This polar form can also be written as follow			$Z=[r; \alpha]$	
	$Z=[r; \alpha]$			Exponential form of a non-zero	
	Exponential form of a non-zero complex			<u>complex_number</u>	
	number			let $Z=[r; \alpha]$ be a complex number,	
	let $Z=[r; \alpha]$ be a complex number,			the exponential form of z is $\mathbf{z} = re^{i\alpha}$	
	the exponential form of z is $\mathbf{z} = re^{i\alpha}$			<u>Remark</u>	
	<u>Remark</u>			If $z = a + ib$, then	
	If $z = a + ib$, then			$ z = \sqrt{a^2 + b^2}$	
	$ z = \sqrt{a^2 + b^2}$ ($a + b$)			$z = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} + i \frac{b}{\sqrt{a^2 + b^2}} \right)$	
	$2 = \sqrt{a^2 + b^2} \left(\frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2}} + i \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2}} \right)$			By definition	
	By definition			$Cos\alpha = \frac{a}{\sqrt{a^2 + b^2}}$ and $sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$	
	$Cos\alpha = \frac{a}{\sqrt{a^2 + b^2}}$ and $sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$			example :	
	example :			give the polar and exponential form of the	
	give the polar and exponential form of the			following complex numbers:	
	following complex numbers:			$z_1 = 1 + i\sqrt{3}$ and $z_2 = 1 + i$	
	$z_1 = 1 + i\sqrt{3}$ and $z_2 = 1 + i$			Resolution	
	Resolution			$ z_1 = 2$	
	$ z_1 = 2$			$(1 \sqrt{3})$	
	$z = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{\sqrt{3}}\right)$			$z_1 = 2\left(\frac{1}{2} + i\frac{1}{2}\right)$	
	$2^{2_1} - 2(2^{+1}, 2)$			$Cos\alpha = \frac{1}{2}$ and $sin \alpha = \frac{\sqrt{3}}{2}$ implies that $\alpha = \frac{\pi}{2}$	
	$Cos\alpha = \frac{1}{2}$ and $sin \alpha = \frac{\sqrt{3}}{2}$ implies that $\alpha = \frac{\pi}{3}$			Then the polar form of	
	Then the polar form of				







Stages /	Teaching /Learning act	ivities	Teacher's	Learners'	Learning Points		Observations
Duration			Activities	Activities			
	$z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\right)$	$\left(\frac{\pi}{3}\right) = \left[2; \frac{\pi}{3}\right]_{\pi}$			$z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\right)$	$\left(\frac{\pi}{3}\right) = \left[2; \frac{\pi}{3}\right]_{\pi}$	
	its exponential form	s $Z_1 = 2e^{\frac{i}{3}}$			its exponential form	is $Z_1 = 2e^{\frac{i}{3}}$	
	$ z_2 = \sqrt{2}; z_2 = \sqrt{2} \left(\frac{1}{2} \right)$	$\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$			$ z_2 = \sqrt{2}; z_2 = \sqrt{2} \left(\frac{1}{2} \right)$	$\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$	
	$Cos\alpha = \frac{\sqrt{2}}{2}$ and $sin\alpha =$	$\frac{\sqrt{2}}{2}$ implies that $\alpha = \frac{\pi}{4}$			$Cos\alpha = \frac{\sqrt{2}}{2}$ and $sin\alpha =$	$=\frac{\sqrt{2}}{2}$ implies that $\alpha = \frac{\pi}{4}$	
	Then the polar form of	of			Then the polar form	of	
	$z_2 = \sqrt{2} \left(\cos \frac{\pi}{4} + i s i \right)$	$n\frac{\pi}{4}$ = $\left[\sqrt{2};\frac{\pi}{4}\right]$			$z_2 = \sqrt{2} \left(\cos \frac{\pi}{4} + isi \right)$	$n\frac{\pi}{4} = \left[\sqrt{2}; \frac{\pi}{4}\right]$	
	its exponential form	s $Z_2 = \sqrt{2}e^{i\frac{\pi}{4}}$			its exponential form	is $Z_2 = \sqrt{2}e^{i\frac{\pi}{4}}$	
	Properties				Properties		
	Let $Z=[r;\alpha] = re^{i\alpha}$ ar	d $z'=[r'; \alpha'] = r'e^{i\alpha'}$			Let $Z=[r;\alpha] = re^{i\alpha}$ and	nd $z'=[r'; \alpha'] = r'e^{i\alpha'}$	
	be two complex numbers;				be two complex numbers;		
	Polar form	Exponential form			Polar form	Exponential form	
	$\bar{z} = [r; -\alpha]$	$\bar{z} = re^{-i\alpha}$			$\bar{z} = [r; -\alpha]$	$\bar{z} = re^{-i\alpha}$	
	$-z=[r;-\alpha]$	$-z = re^{i(\alpha + \pi)}$			$-z=[r;-\alpha]$	$-z = re^{i(\alpha + \pi)}$	
	$z \times z' = [rr'; \alpha + \alpha']$	$z \times z' = rr' e^{i(\alpha + \alpha')}$			$z \times z' = [rr'; \alpha + \alpha']$	$z \times z' = rr'e^{i(\alpha + \alpha')}$	
	$\frac{1}{z} = \left[\frac{1}{r}; -\alpha\right]$	$\frac{1}{z} = \frac{1}{r}e^{-i\alpha}$			$\frac{1}{z} = \left[\frac{1}{r}; -\alpha\right]$	$\frac{1}{z} = \frac{1}{r}e^{-i\alpha}$	
	$\frac{z}{z'} = \left[\frac{r}{r'}; \alpha - \alpha'\right]$	$\frac{z}{z'} = \frac{r}{r'} e^{i(\alpha - \alpha')}$			$\frac{z}{z'} = \left[\frac{r}{r'}; \alpha - \alpha'\right]$	$\frac{z}{z'} = \frac{r}{r'} e^{i(\alpha - \alpha')}$	
	Remark				Remark		
	From the properties w	ve can deduce that :			From the properties v	we can deduce that :	







Stages /	Teaching /Learning activities	Teacher's	Learners'	Learning Points	Observations
Duration		Activities	Activities		
	arg(zz') = arg(z) + arg(z')			arg(zz') = arg(z) + arg(z')	
	$arg(\frac{z}{z}) = arg(z) - arg(z')$			$arg(\frac{z}{z}) = arg(z) - arg(z')$	
	Eversie	Distates the	Taka dayan	Eventies	
F	Exercise 1) Determine the loci of points $M(\pi)$ such that	Dictates the	the eversion	Exercise 1) Determine the last of points $M(\pi)$ such that	
Exercises of	The formula in the later of points $M(Z)$ such that	exercise	and do it	The definition of points $M(Z)$ such that	
Application	z - 1 + i = 1 + 3i		and do it	z - 1 + i = 1 + 3i	
(10mins)	z + 2 - 3i = z + 1 + i			z + 2 - 3i = z + 1 + i	
	2)Given that $z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2}$ and $z_2 = 1 - i$			2)Given that $z_1 = \frac{\sqrt{6-i\sqrt{2}}}{2}$ and $z_2 = 1 - i$	
	a)Determine the modulus and argument of z_1 and z_2			a)Determine the modulus and argument of z_1 and z_2	
	b) Give the algebraic ,polar and exponential form of the			b) Give the algebraic ,polar and exponential form of the	
	quotient $\frac{z_1}{z_1}$			quotient $\frac{z_1}{z_1}$	
	z_2 π , π			z_2 π , π	
	c) Deduce the values of $\cos \frac{\pi}{12}$ and $\sin \frac{\pi}{12}$.			c) Deduce the values of $\cos \frac{\pi}{12}$ and $\sin \frac{\pi}{12}$.	
	Bilingual game			Bilingual game	
Conclusion	Give the equivalence of the following words in French:	Copies	Сору	Give the equivalence of the following words in French:	
(5mins)	modulus; argument; polar form.	questions on	questions in	modulus; argument; polar form.	
	Home work	the board	their note	Home work	
			books		
	Announcement of the next lesson.			Announcement of the next lesson.	
	The next lesson will be on the square root of a complex			The next lesson will be on the square root of a	
	number and solutions of quadratic equations.			complex number and solutions of quadratic	
				equations	