SAMPLE LESSON: MATHEMATICS

Class: Upper Sixth Mathematics

Module: Plane Geometry and solid Figures	**TOPIC:** Complex Numbers
Title of Lesson: n^{th} root of a non-zero complex number, quadratic equation and De Moivre's Theorem.	**Duration of Lesson:** 120mins

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Module 4 : **PLANE GEOMETRY AND SOLID FIGURES**
Topic: Complex Numbers
Lesson: n^{th} root of a non-zero complex number, quadratic equation and De Moivre's Theorem.

Objectives: At the end of this lesson, the learners should be able to:

1) Determine the algebraic form of the square root of a non-zero complex number;
2) Find the n^{th} root of a non-zero complex number;
3) Find solutions of a quadratic equation in $\mathbb{C}$ and also find the roots of polynomial equations with real coefficients;
4) Use De Moivre's Theorem to express
   i) $\sin n\theta, \cos n\theta$ and $\tan n\theta$ in terms of powers of $\sin\theta, \cos\theta$ and $\tan\theta$
   ii) Power of sine or cosine of angles in terms of sine and cosine of multiple angles.

Key question:
How can we solve a polynomial equation in $\mathbb{C}$?
Prerequisite knowledge:

- ✓ Verify whether students can calculate the modulus of a complex number.
- ✓ Verify whether students can solve simultaneous linear equations.

Motivation: The study of complex numbers comes to reinforce our knowledge and skills necessary to study plane geometry.

Didactic materials

Chalk colour chalk, Chalkboard, ruler and set square.

REFERENCES
- EWANE ROLAND ALUNGE. Advanced Level Pure Mathematics Made Easy First Edition.
-Pure Mathematics With Mechanics Teaching Syllabuses(January 2020)

Stages/Duration	Teaching/Learning activities	Teacher's Activities	Learners' Activities	Learning Points	Observations
Introduction **(5mins**	**Verification of Pre-requisites** **Exercise** a) Calculate the modulus of the complex number $z = 4 - 7i$ b) Solve the simultaneous linear equation: $\begin{cases} x + 2y = 2 \\ x - 2y = -4 \end{cases}$	-Copies questions on the board -Calls students to the board	-Solve on the board as called by the teacher, while the others follow up keenly	**Verification of Pre-requisites** **Exercise** a) Calculate the modulus of the complex number $z = 4 - 7i$ b) Solve the simultaneous linear equation: $\begin{cases} x + 2y = 2 \\ x - 2y = -4 \end{cases}$	
Lesson Development and Summary **(100mins)**	**Activity 1** Given a complex number $z = 3 - 4i$ 1)Calculate $(-2 + i)^2$.what can we say about z and $b = -2 + i$ 2)Let $\delta = x + iy$ such that $\delta^2 = z$. a)What can we say about δ and z b)Give a relationship between $\|\delta^2\|$ and $\|z\|$ c) Using the previous information ,find the values (x_1,y_1) and (x_2,y_2) of (x, y). d) Consider $\delta_1 = x_1 + iy_1$ and $\delta_2 = x_2 + iy_2$ Calculate ${\delta_1}^2$, ${\delta_2}^2$ and conclude. **Resolution** 1)we easily verify that $(-2 + i)^2 = 3 - 4i$, We can say that b is a square root of z. 2)a) We can say that δ is a square root of z. b) The relationship between $\|\delta^2\|$ and $\|z\|$ is $\|\delta^2\|$ =$\|z\|$ because $\delta^2 = z$. c) Let's find the values (x_1,y_1) and (x_2,y_2) of (x, y). We have	-Copies activity on the chalkboard -Instructs students to copy in their notebooks and allows them 10 minutes to research	-Follow the instructions and carry out the activity while interacting with each other	**Activity 1** Given a complex number $z = 3 - 4i$ 1)Calculate $(-2 + i)^2$.what can we say about z and $b = -2 + i$ 2)Let $\delta = x + iy$ such that $\delta^2 = z$. a)What can we say about δ and z b)Give a relationship between $\|\delta^2\|$ and $\|z\|$ c) Using the previous information ,find the values (x_1,y_1) and (x_2,y_2) of (x, y). d) Consider $\delta_1 = x_1 + iy_1$ and $\delta_2 = x_2 + iy_2$ Calculate ${\delta_1}^2$, ${\delta_2}^2$ and conclude. **Resolution** 1)we easily verify that $(-2 + i)^2 = 3 - 4i$, We can say that b is a square root of z. 2)a) We can say that δ is a square root of z. b) The relationship between $\|\delta^2\|$ and $\|z\|$ is $\|\delta^2\|$ =$\|z\|$ because $\delta^2 = z$. c) Let's find the values (x_1,y_1) and (x_2,y_2) of (x, y). We have	

	$\lvert\delta^2\rvert = \lvert z\rvert \leftrightarrow x^2+y^2=5$ $\delta^2 = z \leftrightarrow x^2-y^2+i2xy = 3-4i$ By identification, $x^2-y^2=3$ and $2xy=-4$ Then, $\begin{cases} 2xy=-4 \textbf{ is called sign equation} \\ x^2+y^2=5 \\ x^2-y^2=3 \end{cases}$ $\begin{cases} x^2+y^2=5 \\ x^2-y^2=3 \end{cases}$ then $y^2=1$ and $y=1$ or $y=-1$ $x^2=4$ and $x=2$ or $x=-2$ $2xy=-4$ implies that x and y should be opposite in sign If $y=1$ then $x=-2$, $(x_1, y_1)=(-2;1)$ If $y=-1$ then $x=2$, $(x_2, y_2)=(2;-1)$ d) $\delta_1=-2+i$ and $\delta_2=2-i$ ${\delta_1}^2=(-2+i)^2=3-4i$, ${\delta_2}^2=(2-i)^2=3-4i$ **Conclusion**	-Copies notes on the board -Explains concepts	-Copy notes in their books	$\lvert\delta^2\rvert = \lvert z\rvert \leftrightarrow x^2+y^2=5$ $\delta^2 = z \leftrightarrow x^2-y^2+i2xy = 3-4i$ By identification, $x^2-y^2=3$ and $2xy=-4$ Then, $\begin{cases} 2xy=-4 \textbf{ is called sign equation} \\ x^2+y^2=5 \\ x^2-y^2=3 \end{cases}$ $\begin{cases} x^2+y^2=5 \\ x^2-y^2=3 \end{cases}$ then $y^2=1$ and $y=1$ or $y=-1$ $x^2=4$ and $x=2$ or $x=-2$ $2xy=-4$ implies that x and y should be opposite in sign If $y=1$ then $x=-2$, $(x_1, y_1)=(-2;1)$ If $y=-1$ then $x=2$, $(x_2, y_2)=(2;-1)$ d) $\delta_1=-2+i$ and $\delta_2=2-i$ ${\delta_1}^2=(-2+i)^2=3-4i$, ${\delta_2}^2=(2-i)^2=3-4i$ **Conclusion**	
Exercises of Application (10mins)	<u>**Exercise**</u> 1) Determine the fifth roots of the complex number z in the following cases. a) z is unity b) $z=-1+i\sqrt{3}$ 2)Given the following quadratic equation: $(E): z^2+(2+3i)z-2(1-2i)=0$ a) Calculate the discriminant Δ of the equation. b) Find the algebraic form of the square roots of Δ. c) Deduce the solutions of (E). 3) consider the polynomial p define by $p(z) = z^3-2z^2-3z+10$ for any complex number z.	Dictates the exercise	Take down the exercise and do it	<u>**Exercise**</u> 1) Determine the fifth roots of the complex number z in the following cases. a) z is unity b) $z=-1+i\sqrt{3}$ 2)Given the following quadratic equation: $(E): z^2+(2+3i)z-2(1-2i)=0$ a) Calculate the discriminant Δ of the equation. b) Find the algebraic form of the square roots of Δ. c) Deduce the solutions of (E). 3) consider the polynomial p define by $p(z) = z^3-2z^2-3z+10$ for any complex number	

	a)verify that -2 is a root of p. b)Find the values of a,b and c such that $p(z) = (z+2)(az^2+bz+c)$ c)Solve the equation $p(z) = 0$ in $\mathbb{C}$. 4)a)Using De Moivre's Theorem,show that: $\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{1999} = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$ b)Expand the expression,$(cosx + isinx)^3$ and express $cos\ 3x$ and $sin3x$ in terms of $cosx$ and $sinx$.			z. a)verify that -2 is a root of p. b)Find the values of a,b and c such that $p(z) = (z+2)(az^2+bz+c)$ c)Solve the equation $p(z) = 0$ in $\mathbb{C}$. 4)a)Using De Moivre's Theorem,show that: $\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{1999} = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$ b)Expand the expression,$(cosx + isinx)^3$ and express $cos\ 3x$ and $sin3x$ in terms of $cosx$ and $sinx$.	
Conclusion **(5mins)**	**Bilingual game** Give the equivalence of the following words in French: unity; square roots; De Moivre's Theorem. **Home work** **Announcement of the next lesson**. The next lesson will be on revision questions on complex numbers.	Copies questions on the board	Copy questions in their note books	**Bilingual game** Give the equivalence of the following words in French: unity; square roots; De Moivre's Theorem. **Home work** **Announcement of the next lesson**. The next lesson will be on revision questions on complex numbers.	