SAMPLE LESSON: MATHEMATICS

Class: Upper Sixth Mathematics

Module: PLANE GEOMETRY AND SOLID FIGURES **TOPIC:** Complex Numbers

Title of Lesson: Introduction to the complex number system and its properties. **Duration of Lesson:** 120mins

Name of Authors: Inspectorate of Pedagogy/Sciences for the Far North Region

Module 4 : **PLANE GEOMETRY AND SOLID FIGURES**

Topic: Complex Numbers

Lesson: Introduction to the complex number system and its properties.

Objectives: At the end of this lesson, the learners should be able to:

1) Identify a complex number
2) Find the conjugate of a complex number
3) Represent the image of a complex number on an Argand Diagram
4) Add, multiply and divide two complex numbers.

Key question:

What is a complex number?

Prerequisite knowledge:

- ✓ Verify whether students can identify the various sets of numbers.
- ✓ Verify whether students can solve quadratic equations in $\mathbb{R}$.

Motivation: The study of complex numbers comes to reinforce our knowledge and skills necessary to study plane geometry.

Didactic materials

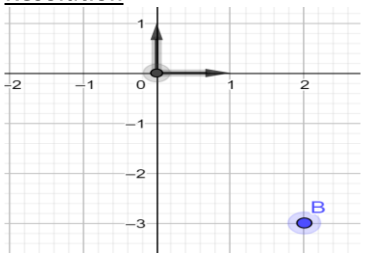
Chalk, colour chalk, Chalkboard, ruler and set square.

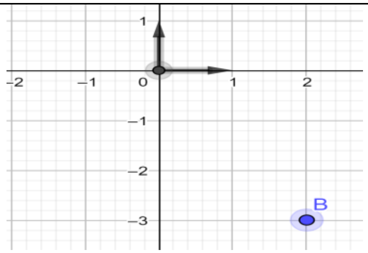
REFERENCES

- EWANE ROLAND ALUNGE. Advanced Level Pure Mathematics Made Easy First Edition.

- Pure Mathematics With Mechanics Teaching Syllabuses (January 2020)

Stages/ Duration	Teaching/Learning activities	Teacher's Activities	Learners' Activities	LEARNING POINTS	Observation
Introduction **(5mins)**	**Verification of Pre-requisites** **Exercise** a)Given the sets $\mathbb{D}$;$\mathbb{N}$;$\mathbb{Q}$;$\mathbb{R}$; $\mathbb{Z}$, rearrange them using the symbol of inclusion "⊂" b)Solve in $\mathbb{N}$ the following equation. $x^2-25=0$.	-Copies questions on the board -Calls students to the board	-Solve on the board as called by the teacher,	**Verification of Pre-requisites** **Exercise** a)Given the sets $\mathbb{D}$;$\mathbb{N}$;$\mathbb{Q}$;$\mathbb{R}$; $\mathbb{Z}$, rearrange them using the symbol of inclusion "⊂" b)Solve in $\mathbb{N}$ the following equation. $x^2-25=0$.	
Lesson Development and Summary **(100mins)**	**Activity** Solve in $\mathbb{R}$ the following equations 1) $x^2-144=0$; 2) $x^2+169=0$ Solution 1) $x^2-144=0 \leftrightarrow x=12\ or\ x=-12$ S=$\{12;-12\}$ 2) $x^2+169=0 \leftrightarrow x^2=-169$ impossible It is impossible to solve this second equation in IR. We should admit the existence of a set bigger than IR which contains a number i such that $i^2=-1$ Then $x^2=i^2 13^2$ And $x=13i$ or $x=-13i$ i will be called imaginary number and this new set will be called set of complex numbers and denoted $\mathbb{C}$. **Definition** A complex number is a number which is written in the form $z=a+ib$ where $(a;b)\in IR^2$ and $i^2=-1$	-Copies activity on the chalkboard -Instructs students to copy in their notebooks and allows them 10 minutes to research -Copies notes on the board -Explains concepts	-Follow the instructions and carry out the activity while interacting with each other -Copy notes in their books	**Activity** Solve in $\mathbb{R}$ the following equations 1) $x^2-144=0$; 2) $x^2+169=0$ Solution 3) $x^2-144=0 \leftrightarrow x=12\ or\ x=-12$ S=$\{12;-12\}$ 4) $x^2+169=0 \leftrightarrow x^2=-169$ impossible It is impossible to solve this second equation in IR. We should admit the existence of a set bigger than IR which contains a number i such that $i^2=-1$ Then $x^2=i^2 13^2$ And $x=13i$ or $x=-13i$ i will be called imaginary number and this new set will be called set of complex numbers and denoted $\mathbb{C}$. **Definition** A complex number is a number which is written in	Ensure logical presentation of work Be rigorous on representatio n

	-The real number a is called the real part of z and denoted $R(z)$. - The real number b is called the imaginary part of z and denoted $Im(z)$. $z = a + ib$ is called the algebraic form of the complex number z. -The set of complex numbers is denoted $\mathbb{C}$. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{D} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ **Example** $z = -4 + 5i$ $z = \frac{2}{5} - 4i$ **Graphic representation of a complex number on an Argand Diagram(complex plane)** For every complex number $z = a + ib$ we can associate a point $M(a;b)$ of the plane. Reciprocally for every point $M(a;b)$ we can always assign a complex number $z = a + ib$ z is called the affix of the point M and M is called the image of the complex number z. **Example** Represent on an Argand Diagram The image B of the complex number $z = 2 - 3i$ **Resolution** 	-Dictates notes Solves the examples one after the other while questioning the students	Copy notes in their notebooks Follow up as the teacher explains, asking and answering questions where necessary	the form $z = a + ib$ where $(a;b) \in IR^2$ and $i^2 = -1$ -The real number a is called the real part of z and denoted $R(z)$. - The real number b is called the imaginary part of z and denoted $Im(z)$. $z = a + ib$ is called the algebraic form of the complex number z. -The set of complex numbers is denoted $\mathbb{C}$. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{D} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ **Example** $z = -4 + 5i$ $z = \frac{2}{5} - 4i$ **Graphic representation of a complex number on an Argand Diagram(complex plane)** For every complex number $z = a + ib$ we can associate a point $M(a;b)$ of the plane. Reciprocally for every point $M(a;b)$ we can always assign a complex number $z = a + ib$ z is called the affix of the point M and M is called the image of the complex number z. **Example** Represent on an Argand Diagram The image B of the complex number $z = 2 - 3i$ **Resolution**	

	Definition The complex number define by $a - ib$ is called the complex conjugate of the complex number $z = a + ib$ and denoted by $\bar{z}$ or z^* **Example** if $z = -4 + 5i$ then $\bar{z} = -4 - 5i$ if $z' = \frac{2}{5} - 4i$ then $\overline{z'}=\frac{2}{5} + 4i$ **properties (of complex conjugate)** let z and z' be two complex numbers →)$\bar{\bar{z}} = z$ →)$\overline{z + z'} = \bar{z} + \overline{z'}$ →)$\overline{z \times z'} = \bar{z} \times \overline{z'}$ →)$\overline{(z^n)}=(\bar{z})^n\ \forall n \in \mathbb{N}$ →)$R_e(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$ **Properties** (equality, sum, multiplication and quotient of complex numbers) Given two complex numbers $z = a + ib$ and $z' = a' + ib'$ →)Equality of two complex numbers $z = z'$ if and only if $a = a'$ and $b = b'$ →)sum of two complex numbers $z + z' = (a + a') + i(b + b')$ →)Product of two complex numbers $z \times z' = aa' - bb' + i(a'b + b'a)$ *Remark* $(a + ib)^2= a^2 - b^2 + i2ab$ $(a - ib)^2= a^2 - b^2 - i2ab$ $(a + ib)(a - ib) = a^2 + b^2$ $z \times z' = 0$ if and only if $z = 0$ or $z' = 0$			 **Definition** The complex number define by $a - ib$ is called the complex conjugate of the complex number $z = a + ib$ and denoted by $\bar{z}$ or z^* **Example** if $z = -4 + 5i$ then $\bar{z} = -4 - 5i$ if $z' = \frac{2}{5} - 4i$ then $\overline{z'}=\frac{2}{5} + 4i$ **properties (of complex conjugate)** let z and z' be two complex numbers →)$\bar{\bar{z}} = z$ →)$\overline{z + z'} = \bar{z} + \overline{z'}$ →)$\overline{z \times z'} = \bar{z} \times \overline{z'}$ →)$\overline{(z^n)}=(\bar{z})^n\ \forall n \in \mathbb{N}$ →)$R_e(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$ **Properties** (equality, sum, multiplication and quotient of complex numbers) Given two complex numbers $z = a + ib$ and $z' = a' + ib'$ →)Equality of two complex numbers $z = z'$ if and only if $a = a'$ and $b = b'$ →)sum of two complex numbers	

	→) Quotient of two complex numbers -Inverse of a complex number If $z \neq 0$ then $\frac{1}{z} = \frac{1}{z} \times \frac{\bar{z}}{\bar{z}} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$ -Quotient of two complex numbers If $z' \neq 0$ $\frac{z}{z'} = z \times \frac{1}{z'}$ $= (a+ib) \times \left(\frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}\right)$ Example Give the algebraic form of the complex number $z = \frac{2+3i}{5+2i}$ Resolution $z = \frac{2+3i}{5+2i} = \frac{2+3i}{5+2i} \times \frac{5-2i}{5-2i} = \frac{16}{29} + \frac{11i}{29}$ Remark Let $z = a + ib$ be a complex number, ➢ z is purely imaginary ↔ $a = 0$ ➢ z is real ↔ $b = 0$ $\frac{z-2i}{z+4} = 0$ be purely imaginary.			$z + z' = (a + a') + i(b + b')$ →)Product of two complex numbers $z \times z' = aa' - bb' + i(a'b + b'a)$ *Remark* $(a+ib)^2 = a^2 - b^2 + i2ab$ $(a-ib)^2 = a^2 - b^2 - i2ab$ $(a+ib)(a-ib) = a^2 + b^2$ $z \times z' = 0$ if and only if $z = 0$ or $z' = 0$ →) Quotient of two complex numbers -Inverse of a complex number If $z \neq 0$ then $\frac{1}{z} = \frac{1}{z} \times \frac{\bar{z}}{\bar{z}} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$ -Quotient of two complex numbers If $z' \neq 0$ $\frac{z}{z'} = z \times \frac{1}{z'}$ $= (a+ib) \times \left(\frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}\right)$ Example Give the algebraic form of the complex number $z = \frac{2+3i}{5+2i}$ Resolution $z = \frac{2+3i}{5+2i} = \frac{2+3i}{5+2i} \times \frac{5-2i}{5-2i} = \frac{16}{29} + \frac{11i}{29}$ Remark Let $z = a + ib$ be a complex number, ➢ z is purely imaginary ↔ $a = 0$ ➢ z is real ↔ $b = 0$ $\frac{z-2i}{z+4} = 0$ be purely imaginary.	

Exercises of Application (10mins)	**<u>Exercise</u>** a)Determine E ,the set of points $M(z)$ such that $\frac{z-2i}{z+4}=0$ be real. b) Determine F, the set of points $M(z)$ such that $\frac{z-2i}{z+4}=0$ be purely imaginary.	Dictates the exercise	Take down the exercise and do it	**<u>Exercise</u>** a)Determine E ,the set of points $M(z)$ such that $\frac{z-2i}{z+4}=0$ be real. b) Determine F, the set of points $M(z)$ such that $\frac{z-2i}{z+4}=0$ be purely imaginary.	
Conclusion (5mins)	**<u>Bilingual game</u>** Give the equivalence of the following words in French: complex number; affix; purely imaginary. **<u>Assignments</u>** Exercise 20.2,page404,Advanced level pure maths made easy. **<u>Announcement of the next lesson</u>**. The next lesson will be on modulus and Argument of a complex number ,polar form of complex number.	Copies questions on the board	Copy questions in their note books	**<u>Bilingual game</u>** Give the equivalence of the following words in French: complex number; affix; purely imaginary. **<u>Assignments</u>** Exercise 20.2,page404,Advanced level pure maths made easy. **<u>Announcement of the next lesson</u>**. The next lesson will be on modulus and Argument of a complex number ,polar form of complex number.	