SAMPLE LESSON: MATHEMATICS

Class: Upper 6

Module: Probability Distribution	**TOPIC:** Random Variable and Discrete Random Variable
Title of Lesson: The expectation of a Random Variable.	**Duration of Lesson:** 50mins

Name of Authors: Group of teachers during training

School: TTP COP **Term**: 2

CLASS: UPPER SIXTH ARTS **Duration**: *50 Mins*

No on Roll: Girls: Boys:

Module:

Topic: *Discrete Random Variables*

Lesson: The expectation of a random variable.

Lesson objective:

At the end of this lesson, learners should be able to calculate and interpret the expectation of any discrete random variable, X.

KEY QUESTION: How can a business man determine the number of items to be bought based on the previous sale?

Perquisite knowledge:

Learners can

- Calculate probabilities of events
- define probability distribution and random variable
- collect data and put them in frequency distribution tables
- calculate the mean of data in frequency distribution tables.

Motivation: The number of students in a school, the number of students to be promoted or dismissed, the amount of money a citizen earns are estimated based on average.

Didactic materials: chalk, ruler, 20 dice.

References: *J CRAWSHAW et al (2004), A concise course in advanced level statistics, fourth edition

*April 2011, Mathematics teaching schemes of work for Advanced level.

<table>
<tr><th>Stages /duration</th><th>Teaching and learning Activities.</th><th>Teacher's Activity</th><th>Learners' Activity</th><th>Learning Points</th></tr>
<tr>
<td>Introduction
(5 minutes)</td>
<td>A/- Verification of pre-requisite knowledge.
1)What is probability distribution?
2)A fair coin is tossed three times. What is the probability of obtaining exactly one head?
3)What is a random variable?
4)Given the data in the table as
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>f</td><td>5</td><td>4</td><td>3</td><td>1</td><td>4</td><td>3</td></tr>
</table>
Use the data to calculate the mean score.

B/-Problem situation.
A man tosses a fair die 240 times and wants to know the mean score. How will you use the results recorded to obtain the mean score?</td>
<td>Asks oral questions

Draws the table on the board and asks the students to calculate the mean.</td>
<td>Respond orally

Calculate and give the answer orally.</td>
<td>Knowledge of probability distribution
Probability of simple events
Knowledge of Random Variable

The mean of the given data is $\frac{16}{5}$</td>
</tr>
<tr>
<td>Lesson Development
30 mins</td>
<td>The man in the problem situation recorded the result as on this table below
<table>
<tr><td>Score, x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Freq, f</td><td>36</td><td>37</td><td>42</td><td>43</td><td>44</td><td>38</td></tr>
</table>
Mean , $\bar{x}$ = $\frac{1\times36+2\times37+3\times42+4\times43+5\times44+6*38}{240}$

=3.57 (2dp)

Mean
$=1\times\frac{36}{240}+2\times\frac{37}{240}+3\times\frac{42}{240}+4\times\frac{43}{240}+5\times\frac{44}{240}+6\times\frac{38}{240}$

*The fractions $\frac{36}{240},\frac{37}{240},\frac{42}{240},\frac{43}{240},\frac{44}{240}$ and $\frac{38}{240}$ are called the relative frequencies.
*The sum of these relative frequencies, $\frac{36}{240},\frac{37}{240},\frac{42}{240},\frac{43}{240},\frac{44}{240}$ and $\frac{38}{240}$ is 1</td>
<td>Tossing a fair die for 240 times is time wasting!
*He gives the 20 dice to the students
*Guides the students to collect data.

*Asks the students to calculate the mean of the data collected. While he supervises learners' activities.

*Asks the students if they would have the same results if the experiment was repeated.</td>
<td>*They toss the 20 dice for 12 times

*They collect data from the 12 tosses.

*They calculate the mean as instructed.

*Answer orally

*Conclude with the teacher.</td>
<td><u>The expectation of a random variable.</u>
Consider a fair die which is tossed 240 times and the results shown on the table below.
<table>
<tr><td>Score, x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Freq,f</td><td>36</td><td>37</td><td>42</td><td>43</td><td>44</td><td>38</td></tr>
</table>
The mean score is calculated as follows:
Mean , $\bar{x}$
$=\frac{1\times36+2\times37+3\times42+4\times43+5\times44+6*38}{240}$

= 3.57 (2dp)

Mean can also be expressed as
$1\times\frac{36}{240}+2\times\frac{37}{240}+3\times\frac{42}{240}+4\times\frac{43}{240}+5\times\frac{44}{240}+6\times\frac{38}{240}$

The fractions $\frac{36}{240},\frac{37}{240},\frac{42}{240},\frac{43}{240},\frac{44}{240}$ and $\frac{38}{240}$ are called the relative frequencies.</td>
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Stages /duration	Teaching and learning Activities.	Teacher's Activity	Learners' Activity	Learning Points
	*Each of these fractions approaches $\frac{1}{6}$ (the probability of obtaining *a 1 or a 2 or a 3 or a 4 or a 5 or a 6 when a fair die is tossed once)* as the number of tosses increases infinitely. *Therefore, in experimental (practical) approach we use the word **mean,** and it is obtained from $\sum score \times relative\ frequency$(for all the scores) While in a theoretical approach, we use the **expected mean** or **expectation** and it is obtained from $\sum score \times probability$ (for all the scores) Hence for any discrete random variable, X, the expectation of X is written as $E(X)$ and it is defined as $\boldsymbol{E(X)}$=$\boldsymbol{\sum_{for\ all\ x}(xP(X=x))}$ Note: *To differentiate between practical and theoretical mean we use $\bar{x}$ for practical mean, and μ for theoretical mean. *$\boldsymbol{\sum_{for\ all\ x}(xP(X=x))}$ means the sum of the product of score and probability of that score.	*The teacher concludes. *He draws a conclusion with the students and deduces the formula for the expectation *puts the first problem on the board and works with the students.	*work with the teacher and copy the notes put on the board by the teacher. *participate by asking or answering questions	The sum of these fractions, $\frac{36}{240}$, $\frac{37}{240}$, $\frac{42}{240}$, $\frac{43}{240}$, $\frac{44}{240}$ and $\frac{38}{240}$ is 1 Each of these fractions approaches $\frac{1}{6}$ (the probability of obtaining *a 1 or a 2 or a 3 or a 4 or a 5 or a 6 when a fair die is tossed once)* as the number of tosses increases infinitely. *Therefore, in experimental (practical) approach we use the word **mean,** and it is obtained from $\sum score \times relative\ frequency$(for all the scores) While in a theoretical approach, we use the **expected mean** or **expectation** and it is obtained from $\sum score \times probability$ (for all the scores) Hence for any discrete random variable, X, the expectation of X is written as $E(X)$ and it is defined as $\boldsymbol{E(X)}$=$\boldsymbol{\sum_{for\ all\ x}(xP(X=x))}$ Note: *To differentiate between practical and theoretical mean we use $\bar{x}$ for practical mean, and μ for theoretical mean. *$\boldsymbol{\sum_{for\ all\ x}(xP(X=x))}$ means the sum of the product of score and probability of that score.
Application Exercises	**Example 1**. An unbiased die is tossed once. Calculate the mean score.	*puts on the questions on the board and marks notebooks	*attempts in their notebooks	**Solution to example 1:** $score$,x: 1, 2, 3, 4, 5, 6 P(X=x): 1/6, 1/6, 1/6, 1/6, 1/6, 1/6

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<td>
Example 2: A random variable Y has probability distribution as shown below.
<table>
<tr><td>y</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>P(Y=y)</td><td>0.3</td><td>0.1</td><td>0.15</td><td>0.4</td><td>0.05</td></tr>
</table>
Find the expectation, E(Y)

Example 3: Find the expected number of sixes when three fair dice are thrown.
</td>
<td></td>
<td></td>
<td>
E(X) = $\sum xP(X=x)$, for all x
= $1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}$ = 3.5
Note: This result is almost the same as the one obtained in the above experiment.

Solution to example 2
E(X)=$\sum yP(Y=y)$ for all y,
=$(-2)\times 0.3+(-1)\times 0.1+0\times 0.15+2\times 0.05$
=-0.2
Solution to example 3
Let M be the number of sixes, then M can take 0 or 1 or 2 or 3.
P(M = 0) = $P(6'\cap 6'\cap 6')$ = $\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}=\frac{125}{216}$
P(M = 1) = $P(6\cap 6'\cap 6')+P(6'\cap 6\cap 6')+P(6'\cap 6'\cap 6)$ = $3\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{75}{216}$
P(M = 2) = $P(6\cap 6\cap 6')+P(6\cap 6'\cap 6)+P(6'\cap 6\cap 6)$ = $3\times\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{15}{216}$
P(M = 3) = $P(6\cap 6\cap 6)=\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$
<table>
<tr><td>m</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(M=m)</td><td>$\frac{125}{216}$</td><td>$\frac{75}{216}$</td><td>$\frac{15}{216}$</td><td>$\frac{3}{216}$</td></tr>
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E(M) = 0.5.
Note: This means that in 500 throws you would expect 0.5×500 = 250 sixes but in practice you may not get 250 sixes.
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<td>Evaluation</td>
<td>1)A random variable X has probability distribution given by</td>
<td></td>
<td></td>
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</tr>
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<td>and

Conclusion</td>
<td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X=x)</td><td>0.5</td><td>0.3</td><td>0.2</td></tr></table>
Calculate E(X)
2)A random variable T can take only two values 1 and x. Given that E(T) = 1.75 and P(T=x)=3P(T=1) . Find the value of x
2) An exam consists of three multiple choice questions each with four suggested answers of which only one is correct. A students decided to choose the answers at random. Let X represent the number of questions he has correct. Find the expectation of X.</td>
<td>Puts the question on the board.</td>
<td>Copy in their notebooks.</td>
<td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X=x)</td><td>0.5</td><td>0.3</td><td>0.2</td></tr></table>
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