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## SAMPLE LESSON: MATHEMATICS

Class: Lower Sixth

**Title of Module:** Plane Geometry

**Title of Chapter:** Derivatives

**Title of Lesson:** Differentiation of Implicit functions

**Duration of Lesson:** 100mins

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## MATHEMATICS LESSON

**SCHOOL:** St Benedict Bilingual College Mvolye

**Term:** 2

**CLASS:** Upper Sixth; **Duration:** 100 minutes ;

**No. on Roll:** \_\_\_\_\_; Boys: \_\_\_\_\_; Girls: \_\_\_\_\_

**MODULE 16:** Calculus I

**TOPIC:** Differentiation

**Lesson:** Differentiation of Implicit Functions Angle Theorems

**Lesson Objectives:** At the end of the lesson, the learners should be able to:

- Identify and differentiate implicit functions
- State real life situations where the knowledge of implicit functions is applied

**Prerequisite knowledge:** Students possess basic knowledge on

Differentiation including chain and product rule, quotient rule as well as differentiation of trigonometric functions.

**Motivation:** Implicit differentiation is used in many areas of sciences like engineering.

### REFERENCES

- A/L Mathematics Teaching Syllabus by Ministry of Secondary Education Cameroon / GCE Board
- Pure Maths by Anucam, 2010 Core Course;
- A/I Maths Made Easy by Ewane 2017



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Stages/Duration	Teaching/Learning Activities	Teacher's Activities	Learners' Activities	Learning Points
<b>Introduction</b>  <b>15 mins</b>        <b>Problem Situation</b>	<b>Motivation and verification of pre-requisite knowledge;</b> Find $dy/dx$ for each of the following i). $y = x^3$ ii). $y = (2x + 1)^5$  iii) $y = \sin(2x)$ iv) $y = x^3 \cos x$  v) $2xy - x^2 + y^2 = 5$	-corrects assignments with students - gives exercises on the board. -organizes the class into groups. -guides students into discovering that they need to go beyond the previous knowledge	-Students go to the board and solve the assignment. -Students sit in groups. -  -Student discuss among themselves to find out the way forward.	<b>Motivation and verification of pre-requisite knowledge;</b> Find $dy/dx$ for each of the following i). $y = x^3$ ii). $y = (2x + 1)^5$  iii) $y = \sin(2x)$ iv) $y = x^3 \cos x$  v) $2xy - x^2 + y^2 = 5$
<b>Lesson Development</b>  <b>35 mins</b>	<u>Activity</u> Differentiating $x = y^2$ with respect to $x$ <u>Instructions</u> To differentiate the right hand side let $u = y^2$ and a) Find $\frac{du}{dy}$ b) using $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$ find an expression for $\frac{du}{dx}$ c) Hence find $\frac{d}{dx}(y^2)$ in terms of $y$ .	-guides the students into discovering the principle underlining implicit differentiation.	-Students come to the conclusion of how to use implicit differentiation.	<u>Activity</u> Differentiating $x=y^2$ with respect to $x$ <u>Instructions</u> To differentiate the right hand side let $u = y^2$ and a) Find $\frac{du}{dy}$ b) using $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$ find an expression for $\frac{du}{dx}$ c) Hence find $\frac{d}{dx}(y^2)$ in terms of $y$ .
<b>Summary</b>  <b>5mins</b>		-explains -writes summary on the board.	- listen	$x = y^2$ $\frac{dx}{dx} = \frac{d}{dx}(y^2)$ $1 = 2y \frac{dy}{dx}$ <b>Definition;</b> A function can be explicit or implicit.

Stages/Duration	Teaching/Learning Activities	Teacher's Activities	Learners' Activities	Learning Points
		-ask questions	-copy	<p>Explicit functions can be written in the form <math>y = f(x)</math> eg <math>y = x^2 + 2y</math>. and <math>y = \sqrt{x + 1}</math> etc</p> <p>While implicit functions are not easily written in the form <math>y = f(x)</math> eg <math>x^2 + y^2 + 2xy = 0</math></p> <p>In the above activity, <math>y^2</math> is differentiated w.r.t <math>y</math> to have <math>2y</math> and the result is multiplied by <math>\frac{dy}{dx}</math>. <b>This process is called IMPLICIT DIFFERENTIATION</b></p>
<p><b>Application Exercises</b></p> <p><b>20mins</b></p>	<p><b>Exercise 1;</b> Find <math>dy/dx</math> in the following equations;</p> <p>1) <math>2xy - x^2 - y^2 = 5</math></p> <p>2) <math>x^2 + xy + y^3 = 8</math></p> <p>3) Given that <math>y + x \sin y = 0</math> show that <math>\frac{dy}{dx} = \frac{-\sin y}{1 + x \cos y}</math></p> <p>4) Given that <math>ye^x = \sin x</math>. Show that <math>\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0</math> is an implicit differentiation</p> <p><b>Practical exercise 2</b></p> <p>Using the balloons given to you, take some time to inflate it and make observations.</p> <p>Given the formula of the volume of a</p>	<p>-writes on the board.</p> <p>-reads</p> <p>-explains</p>	<p>-listen</p> <p>-copy</p> <p>-ask questions</p>	<p><b>Exercise 1;</b> Find <math>dy/dx</math> in the following equations;</p> <p>1) <math>2xy - x^2 - y^2 = 5</math></p> <p>2) <math>x^2 + xy + y^3 = 8</math></p> <p>3) Given that <math>y + x \sin y = 0</math> show that <math>\frac{dy}{dx} = \frac{-\sin y}{1 + x \cos y}</math></p> <p>4) Given that <math>ye^x = \sin x</math>. Show that <math>\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0</math> is an implicit differentiation</p> <p><b>Practical exercise 2</b></p> <p><math>V = \frac{4}{3}\pi r^3</math></p> <p><math>\frac{dr}{dv} = \frac{3}{12}\pi r^2</math></p>



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	sphere, $V = \frac{4}{3}\pi r^3$ What happens to the radius when the volume increases?			
<b>Evaluation</b>  <b>15 mins</b>	Ask oral questions  Announcement	Ask oral questions	Answer the questions asked to show proof of lesson mastery	<b>Assignment</b> <b>1)</b> Given that $y^2 + xy - x^2 = 1$ , find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = y = 1$ <b>2)</b> Given that $\sin y = 2\cos x$ , show that $(\frac{dy}{dx})^2 = 1 + 3\sec^2 y$
<b>Conclusion</b>  <b>10mins</b>		-summarizes, write Assignment, and give announcements -roll call. -signs records of work.	- listen  -ask questions Answer present or absent	