



**AIMS** African Institute for  
Mathematical Sciences  
NEXT EINSTEIN INITIATIVE



## SAMPLE LESSON: MATHEMATICS

Class: Lower Sixth

**Title of Module:** Plane Geometry

**Title of Chapter:** Derivatives

**Title of Lesson:** Differentiation of Implicit functions

**Duration of Lesson:** 100mins

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## MATHEMATICS LESSON

**SCHOOL:** St Benedict Bilingual College Mvolye

**Term:** 2

**CLASS:** Upper Sixth; **Duration:** 100 minutes ;

**No. on Roll:** \_\_\_\_\_; Boys: \_\_\_\_\_; Girls: \_\_\_\_\_

**MODULE 16:** Calculus I

**TOPIC:** Differentiation

**Lesson:** Differentiation of Implicit Functions Angle Theorems

**Lesson Objectives:** At the end of the lesson, the learners should be able to:

- Identify and differentiate implicit functions
- State real life situations where the knowledge of implicit functions is applied

**Prerequisite knowledge:** Students possess basic knowledge on

Differentiation including chain and product rule, quotient rule as well as differentiation of trigonometric functions.

**Motivation:** Implicit differentiation is used in many areas of sciences like engineering.

### REFERENCES

- A/L Mathematics Teaching Syllabus by Ministry of Secondary Education Cameroon / GCE Board
- Pure Maths by Anucam, 2010 Core Course;
- A/I Maths Made Easy by Ewane 2017

| Stages/Duration   | Teaching/Learning Activities   | Teacher's Activities  | Learners' Activities   | Learning Points  |
|---|--|---|--|--|
| <b>Introduction</b><br><br><b>15 mins</b><br><br><b>Problem Situation</b> | <b>Motivation and verification of pre-requisite knowledge;</b><br>Find $dy/dx$ for each of the following<br>i). $y = x^3$<br>ii). $y = (2x + 1)^5$<br><br>iii) $y = \sin(2x)$<br>iv) $y = x^3 \cos x$<br><br>v) $2xy - x^2 + y^2 = 5$  | -corrects assignments with students<br>- gives exercises on the board.<br>-organizes the class into groups.<br>-guides students into discovering that they need to go beyond the previous knowledge | -Students go to the board and solve the assignment.<br>-Students sit in groups.<br>-<br>-Student discuss among themselves to find out the way forward. | <b>Motivation and verification of pre-requisite knowledge;</b><br>Find $dy/dx$ for each of the following<br>i). $y = x^3$<br>ii). $y = (2x + 1)^5$<br><br>iii) $y = \sin(2x)$<br>iv) $y = x^3 \cos x$<br><br>v) $2xy - x^2 + y^2 = 5$  |
| <b>Lesson Development</b><br><br><b>35 mins</b>                           | <u>Activity</u><br>Differentiating $x = y^2$ with respect to $x$<br><u>Instructions</u><br>To differentiate the right hand side let $u = y^2$ and<br>a) Find $\frac{du}{dy}$<br>b) using $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$ find an expression for $\frac{du}{dx}$<br>c) Hence find $\frac{d}{dx}(y^2)$ in terms of $y$ . | -guides the students into discovering the principle underlining implicit differentiation.   | -Students come to the conclusion of how to use implicit differentiation.   | <u>Activity</u><br>Differentiating $x=y^2$ with respect to $x$<br><u>Instructions</u><br>To differentiate the right hand side let $u = y^2$ and<br>a) Find $\frac{du}{dy}$<br>b) using $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$ find an expression for $\frac{du}{dx}$<br>c) Hence find $\frac{d}{dx}(y^2)$ in terms of $y$ . |
| <b>Summary</b><br><br><b>5mins</b>  |  | -explains<br>-writes summary on the board.  | - listen   | $x = y^2$<br>$\frac{dx}{dx} = \frac{d}{dx}(y^2)$<br>$1 = 2y \frac{dy}{dx}$<br><b>Definition;</b> A function can be explicit or implicit.   |



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|--|---|---|--|---|
|  |   | -ask questions  | -copy  | <p>Explicit functions can be written in the form <math>y= f(x)</math> eg <math>y=x^2 +2y</math>. and <math>y=\sqrt{x + 1}</math> etc</p> <p>While implicit functions are not easily written in the form <math>y= f(x)</math> eg <math>x^2 +y^2 +2xy=0</math></p> <p>In the above activity , <math>y^2</math> is differentiated w.r,t <math>y</math> to have <math>2y</math> and the result is multiplied by <math>\frac{dy}{dx}</math> .<b>This process is called IMPLICIT DIFFERENTIATION</b></p>                                  |
| <p><b>Application Exercises</b></p> <p><b>20mins</b></p> | <p><b>Exercise 1;</b> Find <math>dy/dx</math> in the following equations;</p> <p>1) <math>2xy - x^2 - y^2 = 5</math></p> <p>2) <math>x^2 + xy + y^3 = 8</math></p> <p>3) Given that <math>y + xsiny = 0</math> show that <math>\frac{dy}{dx} = \frac{-siny}{1+xcosy}</math></p> <p>4) Given that <math>ye^x = sinx</math>. Show that <math>\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0</math> is an implicit differentiation</p> <p><b>Practical exercise 2</b></p> <p>Using the balloons given to you, take some time to inflate it and make observations.</p> <p>Given the formula of the volume of a</p> | <p>-writes on the board.</p> <p>-reads</p> <p>-explains</p> | <p>- listen</p> <p>-copy</p> <p>-ask questions</p> | <p><b>Exercise 1;</b> Find <math>dy/dx</math> in the following equations;</p> <p>1) <math>2xy - x^2 - y^2 = 5</math></p> <p>2) <math>x^2 + xy + y^3 = 8</math></p> <p>3) Given that <math>y + xsiny = 0</math> show that <math>\frac{dy}{dx} = \frac{-siny}{1+xcosy}</math></p> <p>4) Given that <math>ye^x = sinx</math>. Show that <math>\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0</math> is an implicit differentiation</p> <p><b>Practical exercise 2</b></p> $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dv} = \frac{3}{12} \pi r^2$ |



| Stages/Duration                     | Teaching/Learning Activities   | Teacher's Activities  | Learners' Activities                                       | Learning Points   |
|-------------------------------------|--|---|--|---|
|                                     | sphere, $V = \frac{4}{3}\pi r^3$ What happens to the radius when the volume increases? |   |  |   |
| <b>Evaluation</b><br><b>15 mins</b> | Ask oral questions<br><br>Announcement   | Ask oral questions  | Answer the questions asked to show proof of lesson mastery | <b>Assignment</b><br><b>1)</b> Given that $y^2 + xy - x^2 = 1$ , find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = y = 1$<br><b>2)</b> Given that $\sin y = 2\cos x$ , show that $(\frac{dy}{dx})^2 = 1 + 3\sec^2 y$ |
| <b>Conclusion</b><br><b>10mins</b>  |  | -summarizes, write Assignment, and give announcements<br>-roll call.<br>-signs records of work. | - listen<br><br>-ask questions<br>Answer present or absent |   |