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## **SAMPLE LESSON: MATHEMATICS**

**Class: Upper Sixth/ Further Mathematics**

**Module:** Geometry II

**TOPIC:** Complex Numbers

**Title of Lesson:** Complex number and Plane Transformations

**Duration of Lesson:** 120mins

**Name of Authors:** Inspectorate of Pedagogy for the Far North Region



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## Module 4 : GEOMETRY II

**Topic:** Complex Numbers

**Lesson:** Complex number and Plane Transformations

**Objectives:** At the end of this lesson, the learners should be able to:

- 1) Determine the complex expression of some transformations: Translation, reflection, rotation and glide reflection ;
- 2) Find the image of points with given affixes under the transformations above.

**Key question:**

How to find and use the complex expression of some plane transformations?

**Prerequisite knowledge:**

- ✓ Verify whether students can give the characteristic elements of some transformations.

**Motivation:** The study of complex numbers comes to reinforce our knowledge and skills necessary to study plane geometry.

**Didactic materials**

Chalk, colour chalk, Chalkboard, ruler and set square.

**REFERENCES**

- EWANE ROLAND ALUNGE. Further Pure Mathematics Made Easy Third Edition.
- FURTHER Mathematics Teaching Syllabuses (January 2020)



Stages/ Duration	Teaching/Learning activities	Teacher's Activities	Learners' Activities	Learning Points	Observation
<b>Introduction (5mins)</b>	<p><b>Verification of Pre-requisites</b></p> <p><b>Exercise</b> what are the characteristic elements of the following transformations: Translation; reflection; rotation and glide symmetry.</p>	<p>-Copies questions on the board</p> <p>-Calls students to the board</p>	<p>-Solve on the board as called by the teacher, while the others follow up keenly</p>	<p><b>Verification of Pre-requisites</b></p> <p><b>Exercise</b> what are the characteristic elements of the following transformations: Translation; reflection; rotation and glide symmetry.</p>	
<b>Lesson Development and Summary (100mins)</b>	<p><b>Activity1</b> Given a vector <math>\vec{v}</math> with affix <math>3 - 4i</math>. a) Find the complex expression of the translation <math>t_{\vec{v}}</math> b) Find the image of the following points: <math>A(1 + 2i), B(-2 + 2i)</math> under the translation <math>t_{\vec{v}}</math>.</p> <p><b>Resolution</b> a) For any point <math>M(z)</math> of the plane <math>\mathcal{P}</math>, <math>t_{\vec{v}}(M) = M' \leftrightarrow \overrightarrow{MM'} = \vec{v}</math> <math>\leftrightarrow z_{\overrightarrow{MM'}} = z_{\vec{v}}</math> <math>\leftrightarrow z_{M'} - z_M = z_{\vec{v}}</math> <math>\leftrightarrow z' - z = 3 - 4i</math> where <math>z'</math> is the affix of <math>M'</math> <math>\leftrightarrow z' = z + 3 - 4i</math> is the complex expression of the translation <math>t_{\vec{v}}</math>. b) let's find the images of A and B.</p>	<p>-Copies activity on the chalkboard</p> <p>-Instructs students to copy in their notebooks and allows them 10 minutes to research</p>	<p>-Follow the instructions and carry out the activity while interacting with each other</p> <p>-Copy notes in their books</p>	<p><b>Activity1</b> Given a vector <math>\vec{v}</math> with affix <math>3 - 4i</math>. a) Find the complex expression of the translation <math>t_{\vec{v}}</math> b) Find the image of the following points: <math>A(1 + 2i), B(-2 + 2i)</math> under the translation <math>t_{\vec{v}}</math>.</p> <p><b>Resolution</b> a) For any point <math>M(z)</math> of the plane <math>\mathcal{P}</math>, <math>t_{\vec{v}}(M) = M' \leftrightarrow \overrightarrow{MM'} = \vec{v}</math> <math>\leftrightarrow z_{\overrightarrow{MM'}} = z_{\vec{v}}</math> <math>\leftrightarrow z_{M'} - z_M = z_{\vec{v}}</math> <math>\leftrightarrow z' - z = 3 - 4i</math> where <math>z'</math> is the affix of <math>M'</math> <math>\leftrightarrow z' = z + 3 - 4i</math> is the complex expression of the translation <math>t_{\vec{v}}</math>. b) let's find the images of A and B.</p>	



Stages/ Duration	Teaching/Learning activities	Teacher's Activities	Learners' Activities	Learning Points	Observation
	<p><math>z_{B'} = z_B + 3 - 4i</math> then <math>z_{B'} = 1 - 2i</math>  <math>z_{A'} = z_A + 3 - 4i</math> then <math>z_{A'} = 4 - 2i</math></p> <p><b>Activity2</b>            Given a point <math>A</math> with affix <math>3 - 4i</math>            a) Find the complex expression of the central symmetry <math>S</math> with center <math>A</math>.            b) Find the image of the following points: <math>k(1 - i), B(-4 + 2i)</math> under the translation <math>S</math>.</p> <p><b>Resolution</b>            a) For any point <math>M(z)</math> of the plane <math>\mathcal{P}</math>  <math>S(M) = M' \iff \overrightarrow{AM'} = -\overrightarrow{AM}</math>  <math>\iff z_{AM'} = -z_{AM}</math>  <math>\iff z_{M'} - z_A = -(z_M - z_A)</math>  <math>\iff z_{M'} = 2z_A - z_M</math>  <math>\iff z' = -z + 2z_A</math> where <math>z'</math> is the affix of <math>M'</math></p> <p>b) let's find the images of A and B.  <math>z_{K'} = 2z_A - z_K = 2(3 - 4i) - (1 - i) = 5 - 7i</math>  <math>z_{B'} = 2z_A - z_B = 2(3 - 4i) - (-4 + 2i) = 10 - 10i</math></p>	<p>-Copies notes on the board</p> <p>-Explains concepts</p> <p>-Dictates notes</p> <p>-Carries out the proves to obtain the various complex expression</p>	<p>Copy notes in their notebooks</p>	<p><math>z_{B'} = z_B + 3 - 4i</math> then <math>z_{B'} = 1 - 2i</math>  <math>z_{A'} = z_A + 3 - 4i</math> then <math>z_{A'} = 4 - 2i</math></p> <p><b>Activity2</b>            Given a point <math>A</math> with affix <math>3 - 4i</math>            a) Find the complex expression of the central symmetry <math>S</math> with center <math>A</math>.            b) Find the image of the following points: <math>k(1 - i), B(-4 + 2i)</math> under the translation <math>S</math>.</p> <p><b>Resolution</b>            a) For any point <math>M(z)</math> of the plane <math>\mathcal{P}</math>  <math>S(M) = M' \iff \overrightarrow{AM'} = -\overrightarrow{AM}</math>  <math>\iff z_{AM'} = -z_{AM}</math>  <math>\iff z_{M'} - z_A = -(z_M - z_A)</math>  <math>\iff z_{M'} = 2z_A - z_M</math>  <math>\iff z' = -z + 2z_A</math> where <math>z'</math> is the affix of <math>M'</math></p> <p>b) let's find the images of A and B.  <math>z_{K'} = 2z_A - z_K = 2(3 - 4i) - (1 - i) = 5 - 7i</math>  <math>z_{B'} = 2z_A - z_B = 2(3 - 4i) - (-4 + 2i) = 10 - 10i</math></p>	



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	<p><b>Properties</b></p> <p>→)If <math>t_{\vec{v}}</math> is a translation ,the complex expression of <math>t_{\vec{v}}</math> is given by:  <math>z' = z + z_{\vec{v}}</math></p> <p>→)If S is a central symmetry with center <math>\Omega</math>, the complex expression of <math>S</math> is given by:  <math>z' = -z + 2z_{\Omega}</math></p> <p>→) If S is a reflection about the X-axis the complex expression of <math>S</math> is given by:  <math>z' = \bar{z}</math></p> <p>→) If S is a reflection about the Y-axis the complex expression of <math>S</math> is given by:  <math>z' = -\bar{z}</math></p> <p>→) If S is a reflection about the line(<math>\Delta</math>) with equation <math>y = b</math> ,the complex expression of <math>S</math> is given by:  <math>z' = \bar{z} + 2bi</math></p> <p>→)If <math>r(\Omega;\theta)</math> is a rotation with center <math>\Omega</math> and angle <math>\theta</math>.  the complex expression of <math>r</math> is given by:  <math>z' = e^{i\theta}z + (1 - e^{i\theta})z_{\Omega}</math></p> <p>→) If <math>h(\Omega, k)</math> is a homothety ,  the complex expression of <math>h</math> is given by:  <math>z' = kz + (1 - k)z_{\Omega}</math></p> <p><b>Definition</b></p>		<p>Follow up as the teacher explains, asking and answering questions where necessary</p>	<p><b>Properties</b></p> <p>→)If <math>t_{\vec{v}}</math> is a translation ,the complex expression of <math>t_{\vec{v}}</math> is given by:  <math>z' = z + z_{\vec{v}}</math></p> <p>→)If S is a central symmetry with center <math>\Omega</math>, the complex expression of <math>S</math> is given by:  <math>z' = -z + 2z_{\Omega}</math></p> <p>→) If S is a reflection about the X-axis the complex expression of <math>S</math> is given by:  <math>z' = \bar{z}</math></p> <p>→) If S is a reflection about the Y-axis the complex expression of <math>S</math> is given by:  <math>z' = -\bar{z}</math></p> <p>→) If S is a reflection about the line(<math>\Delta</math>) with equation <math>y = b</math> ,the complex expression of <math>S</math> is given by:  <math>z' = \bar{z} + 2bi</math></p> <p>→)If <math>r(\Omega;\theta)</math> is a rotation with center <math>\Omega</math> and angle <math>\theta</math>.  the complex expression of <math>r</math> is given by:  <math>z' = e^{i\theta}z + (1 - e^{i\theta})z_{\Omega}</math></p> <p>→) If <math>h(\Omega, k)</math> is a homothety ,  the complex expression of <math>h</math> is given by:  <math>z' = kz + (1 - k)z_{\Omega}</math></p> <p><b>Definition</b></p>	



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	<p>Glide symmetry is a composition of a translation and a orthogonal symmetry where the vector of translation is a direction vector of the axis of symmetry.</p> $f = t_{\vec{v}} \circ S_{\Delta}$ <p>→If <math>f = t_{\vec{v}} \circ S_{\Delta}</math> is a glide symmetry</p> <p>Where <math>(\Delta)</math> has as equation <math>y = b</math> and <math>\vec{v}</math> a direction vector of <math>(\Delta)</math>,the complex expression of <math>f</math> is given by:</p> $z' = \bar{z} + 2bi + Z\vec{v}$			<p>Glide symmetry is a composition of a translation and a orthogonal symmetry where the vector of translation is a direction vector of the axis of symmetry.</p> $f = t_{\vec{v}} \circ S_{\Delta}$ <p>→If <math>f = t_{\vec{v}} \circ S_{\Delta}</math> is a glide symmetry</p> <p>Where <math>(\Delta)</math> has as equation <math>y = b</math> and <math>\vec{v}</math> a direction vector of <math>(\Delta)</math>,the complex expression of <math>f</math> is given by:</p> $z' = \bar{z} + 2bi + Z\vec{v}$	
<p><b>Conclusion (5mins)</b></p>	<p><b>Bilingual game</b> Give the equivalence of the word complex expression in French.</p> <p><b>Home work</b></p> <p><b>Announcement of the next lesson.</b> The next lesson will be on Complex number and similarity</p>	<p>Copies questions on the board</p>	<p>Copy questions in their note books</p>	<p><b>Bilingual game</b> Give the equivalence of the word complex expression in French_</p> <p><b>Assignments</b></p> <p><b>Announcement of the next lesson.</b> The next lesson will be on Complex number and similarity</p>	