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## SAMPLE LESSON: MATHEMATICS

Class: Form 5

**Title of Module:** SOLID FIGURES.

**Title of Chapter:** Solids

**Title of Lesson:** Total surface area and volume of similar solids

**Duration of Lesson:** 60mins



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**SCHOOL:** AIMS TTP COP

**CLASS:** FORM 5

**Term:**

**Date:**

**ENROLMENT:** Boys: ; Girls:

**DURATION :**

**MODULE:** SOLID FIGURES

**TOPIC:** SOLIDS

**LESSON:** Total surface Area and Volume of similar figures

**Rationale:** We live in a 3-Dimensional world and 3-D figures are all around us, in nature or in things produced by man. We equally use these shapes in designing, constructions, packaging, storage etc in real life. As such we are always face with situations to determine how much space something can occupy or how much content can go into a container or better still the capacity of a container.

**Objectives:**

- Find length, surface areas and volumes of similar solid figures using the scale factors
- Find total surface area and volume of solids;
- Calculate unknown dimensions for given solids

**Prerequisite knowledge: Learners can**

- Identify and recognize different solids.
- Use scale factor to establish similar figures or solids
- Calculate total surface area of cones, cuboids, cylinders and prisms,
- Calculate volume of cones, cuboids, cylinders and prisms.

**DIDACTIC MATERIALS:** Graph board, different solids, mathematical instruments and worksheets.

**REFERENCE:**

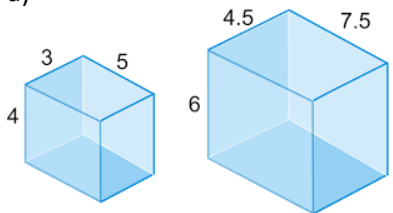
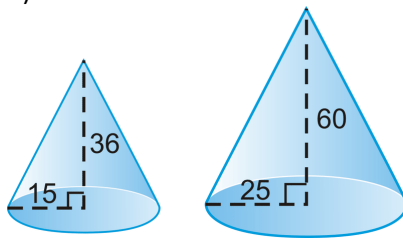
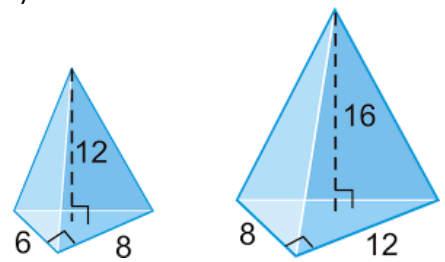
- Mathematics 9, M. J Tipler and J Douglas, 2004, Nelson Thornes Ltd
- Integrated Core Approach, Ordinary Level Mathematics, Piankeh Albert 2011, Third Edition, MB Mbosso Publishers

**Preparation:**

Design, print out and photocopy worksheets for learners depending on the number on class and expected number of groups. Draw out solids for verification of prerequisite of cardboard papers and past up for learners to see.

Gather some common solids around and take to class for recall of shapes thus their formulae for calculation of areas and volumes.



Stages /Duration	Teaching / learning ACTIVITIES	Learner 's Activities	Learning Point	Observation
Introduction	<p><b>Verification of Pre-requisite knowledge</b> Paste up pairs of figures, assign benches to work on pairs. The whole class should not work on all. 1. for each pair of figure, give the name of the solid then Say if each pair of solids are similar. Justify</p> <p>a)</p>  <p>b)</p>  <p>c)</p> 	<p>Each bench works on the exercise given to them and gives the answer.</p> <p>Bench representative gives their answer. This answer is verified by another bench doing the same tasks</p>	<p>For similar figures or shape, corresponding sides are proportional. The ratio of corresponding sides are equal</p> <p>a)The cuboids Ratio of corresponding sides: <math>\frac{4}{6} = \frac{2}{3}</math>; <math>\frac{3}{4.5} = \frac{30}{45} = \frac{2}{3}</math> and <math>\frac{5}{7.5} = \frac{50}{75} = \frac{2}{3}</math> Ratio of corresponding sides are equal thus the cuboids are similar.</p> <p>b) The Cones. Ratio of corresponding sides: <math>\frac{15}{25} = \frac{3}{5}</math> and <math>\frac{36}{60} = \frac{3}{5}</math> The cones are similar.</p> <p>c)Pyramids Ratio of corresponding sides: <math>\frac{6}{8} = \frac{3}{4}</math>; <math>\frac{8}{12} = \frac{2}{3}</math>; <math>\frac{21}{28} = \frac{3}{4}</math> But <math>\frac{3}{4} \neq \frac{2}{3} \therefore</math> solids are not similar</p>	<p>Let the learners work in groups according to their sitting positions</p> <p>All misconceptions must be corrected at this level</p>



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	<p>2.a) Calculate the total surface area of the smaller cuboid in a)</p> <p>b) Calculate the volume of the larger cone in b) .</p>		<p>2)</p> <p>a) cuboid is of sides 4 by 3 by 5 Total surface area will be <math>2(4 \times 3) + 2(4 \times 5) + 2(3 \times 5) = 94</math> cubic units</p> <p>b) Volume of cone: <math>\frac{1}{3}\pi r^2 h</math>  <math>r = 25</math>; height = 60  <math>\therefore \text{volume} = \frac{1}{3}\pi \times 25^2 \times 60</math>  Volume = 39285.7 cubic units to 1dp.</p>	
	<p><b>Problem Situation: Volume of water in a water tank to serve a given population.</b></p> <p>The new Mayor of Down town district has as one of her projects to solve the problem of water shortage in that neighborhood. A study carried out shows that the water tank that supplies water for the area is more than 30 years old. For these years, the population has increased drastically. As such the capacity of the water tank is insufficient to satisfy the daily need of all households. The old tank of radius 5m could hold 1100m<sup>3</sup> of water when full. The mayor tells the contractor to construct a tank that is similar to the old tank and whose radius will be twice the old tank. What will then be the Volume of water when the new tank is full. What will be the height of the new tank?</p>			<p>If typed out distribute to students otherwise read out to them</p>
<p><b>Lesson development (30 mins )</b></p>	<p><b>Learning Activity: Give the pairs of solids whose scale factors were got during verification of pre-requisite.</b></p> <p>1. For each pair of solids, the ratio of corresponding sides are given. Calculate a) the total surface area of each b) the volume of each</p> <p>2. Find the ratio of the area of the smaller solid to the total surface area of the larger solid</p> <p>3. find the ratio of the volume of the smaller solid to the volume of the larger solid</p> <p>4. Complete the table on the worksheet</p>	<p>Students work in groups to answer questions as on the worksheet.</p>	<p><b>Conclusion</b></p> <p>If ratio of the lengths of corresponding sides of similar figure is <math>\frac{a}{b}</math>, <math>b \neq 0</math> also called Scale Factor written as a:b, then</p> <p>i) the ratio of their total areas is <math>\left(\frac{a}{b}\right)^2</math></p> <p>ii) the ratio of their volumes is <math>\left(\frac{a}{b}\right)^3</math></p> <p>In other words "When the dimensions of a solid are multiplied by k, the surface area is</p>	<p>Guide them but without telling them the answers.</p> <p>Listen keenly to their spoken language and</p>



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	<p>5. What is the relationship between the total surface area of the smaller solid and that of the larger solid?</p> <p>6)What is the relationship between the volume of the smaller solid and that of the larger solid?</p> <p>7)What can you say about total surface area of similar solids?</p> <p>8)What can you conclude about volume of similar figures?</p>	<p>Groups present their results.</p> <p>Students together with teacher come out with a conclusion.</p>	<p><i>multiplied by <math>k^2</math> and the volume is multiplied by <math>k^3</math> ."</i></p>	<p>correct them insisting on technical words.</p>
<b>Exercises for Application</b>	<p>Do the following exercises in your note books.</p> <p>1.Go back to the problem situation and find the height of the tank to be constructed.</p> <p>2. Two cubes are similar. The side of the smaller cube is 5cm while the side of the larger cube is 8cm. The surface area of the smaller cube is <math>150\text{cm}^2</math>. What is a) the total surface area of the larger cube. b)the scale factor of their volumes?</p>		<p><b>1.</b>New tank is to be of radius twice the old tank, it means the ratio of the old radius to the new is 1:2. Therefore ratio of volume will be 1:8  Volume of old tank is <math>1100\text{m}^3</math>  <math>\therefore</math> Volume of new tank will be:  <math>1100 \times 8 = 8800\text{m}^3</math>  Vol of old tank <math>v = 1100 = \frac{22}{7}(25)h</math>  <math>\therefore h = \frac{7700}{22 \times 25} = 14\text{m}</math>  Height of New tank will be:  <math>2 \times 14\text{m} = 28\text{m}</math></p> <p><b>2.</b> Scale factor of sides is <math>\frac{5}{8}</math>. Scale factor of area will be <math>\frac{25}{64}</math>  a)So total surface area of larger cube will be <math>150 \times \frac{64}{25} = 1600\text{cm}^3</math></p>	<p>Guide them but without telling them the answers.</p> <p>Insist on orderly presentation of solutions</p>



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	<p>3. M and N are similar triangular pyramids. Their corresponding volumes are 612 m<sup>3</sup> and 1728m<sup>3</sup>. Determine the scale factor of N to M.</p> <p><b>4. Complete each of the tables below:</b></p> <p>a)</p> <table><tr><th>Surface area (SA) of solid A</th><th>Scale factor of dimensions of A to B</th><th>Ratio of Surface Area of A to B</th><th>Surface area of B</th></tr><tr><td>20m<sup>2</sup></td><td>2:3</td><td></td><td></td></tr><tr><td>162m<sup>2</sup></td><td>9:1</td><td></td><td></td></tr><tr><td>1250cm<sup>2</sup></td><td>5:7</td><td></td><td></td></tr></table> <p>b)</p> <table><tr><th>Volume of P</th><th>SF of P to that of Q</th><th>SF of Vol of P to vol of Q</th><th>Volume of Q</th></tr><tr><td>10290cm<sup>3</sup></td><td>7:2</td><td></td><td></td></tr><tr><td>4.096m<sup>3</sup></td><td>1:5</td><td></td><td></td></tr><tr><td>3645m<sup>3</sup></td><td>9:8</td><td></td><td></td></tr></table>	Surface area (SA) of solid A	Scale factor of dimensions of A to B	Ratio of Surface Area of A to B	Surface area of B	20m <sup>2</sup>	2:3			162m <sup>2</sup>	9:1			1250cm <sup>2</sup>	5:7			Volume of P	SF of P to that of Q	SF of Vol of P to vol of Q	Volume of Q	10290cm <sup>3</sup>	7:2			4.096m <sup>3</sup>	1:5			3645m <sup>3</sup>	9:8				<p>b)Scale factor of volumes of cube is <math>\frac{125}{512}</math>.</p> <p>3. Scale factor of their volumes <math>\frac{612}{1728}</math>. Taking cube roots of both numerator and denominator gives <math>\frac{8}{12}</math></p> <p>4.a)</p> <table><tr><th>Surface area (SA) of solid A</th><th>Scale factor of dimensions of A to B</th><th>Ratio of Surface Area of A to B</th><th>Surface area of B</th></tr><tr><td>20m<sup>2</sup></td><td>2:3</td><td>4:9</td><td>45m<sup>2</sup></td></tr><tr><td>162m<sup>2</sup></td><td>9:1</td><td>81:1</td><td>2m<sup>2</sup></td></tr><tr><td>1250cm<sup>2</sup></td><td>5:7</td><td>25:49</td><td>2450cm<sup>2</sup></td></tr></table> <p>b)</p> <table><tr><th>Volume of P</th><th>SF of P to that of Q</th><th>SF of Vol of P to vol of Q</th><th>Volume of Q</th></tr><tr><td>10290cm<sup>3</sup></td><td>7:2</td><td>343:8</td><td>240</td></tr><tr><td>4.096m<sup>3</sup></td><td>1:5</td><td>1:25</td><td>512</td></tr><tr><td>3645m<sup>3</sup></td><td>9:8</td><td>729:512</td><td>2560</td></tr></table>	Surface area (SA) of solid A	Scale factor of dimensions of A to B	Ratio of Surface Area of A to B	Surface area of B	20m <sup>2</sup>	2:3	4:9	45m <sup>2</sup>	162m <sup>2</sup>	9:1	81:1	2m <sup>2</sup>	1250cm <sup>2</sup>	5:7	25:49	2450cm <sup>2</sup>	Volume of P	SF of P to that of Q	SF of Vol of P to vol of Q	Volume of Q	10290cm <sup>3</sup>	7:2	343:8	240	4.096m <sup>3</sup>	1:5	1:25	512	3645m <sup>3</sup>	9:8	729:512	2560	
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Conclusion	<p><b>Home work</b></p> <p>1.The volume of a Rectangular pyramid whose rectangular base are of dimension 4cm and 2.5cm and height 9cm is 30cm<sup>3</sup>. Find the volume of a</p>		<p>1. If both pyramids are similar, then corresponding sides are proportional.</p> <p>Scale factor <math>\frac{4}{8} = \frac{2.5}{5} = \frac{1}{2}</math></p>																																																																	



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	<p>similar pyramid whose base ABCD, is such that AB = 8cm and BC = 5cm?</p> <p>2. A regular square pyramid with base edges of length 10cm and lateral edges of length 12 cm has a volume of <math>\frac{100}{3}\sqrt{94}cm^3</math> is to serve as a model in producing other pyramids. Find a) the total surface area and b) the volume of a pyramid whose sides are three times the dimensions of the model.</p> <p>3.The volume of a sphere whose radius is 10cm is <math>6280cm^3</math> to the nearest whole number. Find the volume of the sphere whose radius is 5cm.</p> <p>4. Two similar pyramids have surface areas <math>200m^2</math> and <math>500m^2</math>. The smaller pyramid has volume of <math>500m^3</math>. Find the volume of the larger pyramid.</p>	Copy home work in the books	<p>If scale factor for dimensions is <math>\frac{1}{2}</math>, then the scale factor for volume will be <math>\frac{1}{8}</math>.</p> <p>Volume of the second pyramid will be: <math>8 \times 30cm^2 = 240cm^3</math></p> <p><b>3. Volume of sphere <math>\frac{4}{3}\pi r^3</math></b> If radius of sphere is 10cm, then its height is 20cm. For the second sphere, the radius is 5cm so the height is 10cm. Scale factor for dimensions is <math>\frac{5}{10} = \frac{10}{20} = \frac{1}{2}</math> Volume of smaller sphere will be <math>\frac{1}{2^3} = \frac{1}{8}</math> of the bigger sphere. <math>Vol = \frac{1}{8} \times 6280 = 785cm^3</math>.</p>	



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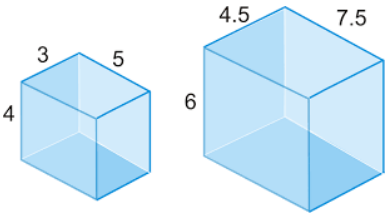
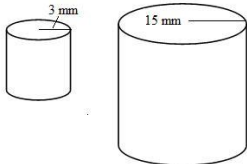
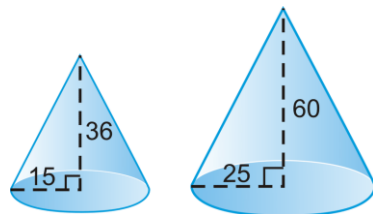
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## Activity worksheet

### Instructions

Consider each pair of figures on the table below.

 <p><b>Pair 2</b></p>	 <p>Height of smaller is 5mm and height of larger is 25mm</p> <p><b>Pair 3</b></p>	 <p><b>Pair 4</b></p>
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1. Say whether the figures are similar or not and why
2. State the scale factor for each pair of similar figures
3. Calculate the total surface area of each solid and write by it
4. Determine the ratio of the areas and write it down
5. Calculate the volume of each solid and write by it
6. Find the ratio of the volumes and write it down
7. Complete the table below:

Pair	Ratio of sides	Ratio of Total Surface Areas	Ratio of Volume	Ratio of area in terms of ratio of sides	Ratio of volume in terms of ratio of sides	Conclusion
1	2:3	940:2115 = 4:9	600:2025 = 8 : 27	$4: 9 = 2^2: 3^2$	$8: 27 = 2^3 : 3^3$	
2	1:5	1056:26400 = 1: 25	501:12525 = 1:25	$1056:26400 = 1:5^2$	$900:123750=1:5^3$	
3	3:5	4950:13750 = 9 : 25	1782:8250 = 27:125	$4950:13750 = 3^2:5^2$	$27:125 = 3^3 : 5^3$	

8. What can you conclude about the area of similar figures?
9. In your own words say something about the volume of similar figures.